Let X be a separable metric space, μ a Borel measure on X, and B some family of open sets of finite positive measure. Define the maximal function of a non-negative integrable function f by

$$Mf(x) = \sup_{R \in B, x \in R} \mu(R)^{-1} \int_R f \, d\mu \, .$$

The weak type L^1 bound for M, i.e., the inequality

$$\mu(\{x \in X : Mf(x) > t\}) \leq Ct^{-1} \int_X f \, d\mu \, ,$$

in various settings is usually derived from some covering selection property, the most general version of which seems to be as follows: There exist constants $c, C \in (0, +\infty)$ such that for every finite family $B_0 \subset B$, there is a subfamily $B_1 \subset B_0$ satisfying $\mu(\cup_{R \in B_1} R) \geq c\mu(\cup_{R \in B_0} R)$ and $\sum_{R \in B_1} \chi_R \leq C$ μ-almost everywhere. We shall show that there cannot be any other reason for a weak type L^1 bound of the above type, namely, that if the weak type bound holds for the maximal function M associated with the family B, then B necessarily has this covering selection property. Time permitting, we'll discuss analogues of this theorem for the similar bounds for M involving $L \log L$ and other Orlicz type expressions on the right-hand side. This is a joint work with Paul Hagelstein and Blanca Radillo-Murguia.