U-adic integers for some Pisot U-numerations.

A Pisot numeration system $U = (u_n)_{n\geq 0}$ for $\mathbb{N}$ is a sequence of natural numbers generated by an integral homogeneous linear recurrence, whose characteristic polynomial is the minimal polynomial of a Pisot number. The simplest example is the Zeckendorf numeration. In joint work with Olivier Carton and Jake Sudbery, we introduce, for each Pisot numeration U satisfying a mild condition, a group $\mathbb{Z}_U$ which is the analogue of the additive group $\mathbb{Z}_p$ of the p-adic integers. It can be viewed as a topological completion of the set of *normalized* U-expansions of natural integers. This construction was worked out by Vershik in the nineties for the Zeckendorf numeration, who showed, via character theory, that $\mathbb{Z}_U$ is a circle.

Our construction of $\mathbb{Z}_U$ is based not on character theory, but on the notion of normalisation of addition. This is the process of converting a component-wise addition of expansions to the normalised form. Our approach does not follow the same strategy as for $\mathbb{Z}_p$ for the following reason. When two base-p expansions are added, the carries propagate to the left. This is reflected in the fact that the p-adic valuation ν_p satisfies the classical inequality $\nu_p(x + y) \geq \min(\nu_p(x), \nu_p(y))$. This property no longer holds for Pisot numerations; carries may propagate both to the left and to the right, as is already the case for the Zeckendorf numeration. For this reason, our construction is based on a pseudo-valuation ν_U which satisfies the weaker inequality $\nu_U(x+y) \geq \min(\nu_U(x), \nu_U(y))-K$ for some constant K; we say that numerations that satisfy this *preserve zeros.* We also equip our group $\mathbb{Z}_U$ with a topology. As a result of the nonstandard propagation of the carries, $\mathbb{Z}_U$ is not a profinite group with a zero dimensional topology. We show

Theorem 1. *Let $U = (u_n)_{n\geqslant 0}$ be a Pisot numeration with standard initial conditions that preserves zeros. Then there are k, ℓ and a continuous group isomorphism $\Phi : \mathbb{Z}_U \to (\mathbb{C}^k \times \mathbb{R}^\ell)/\mathbb{L}$ where $\mathbb{L}$ is a lattice.*

Time permitting, I will discuss how this can be applied to show that some substitution dynamical systems possess *independence sequences*, implying that their Ellis semigroup is nontame, i.e., has cardinality $2^{|\mathbb{R}|}$.