Optimizing the design of a loyalty program

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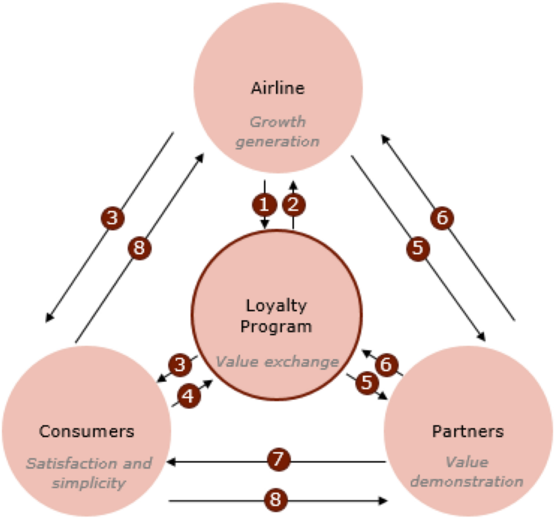
Aeroplan Loyalty Program



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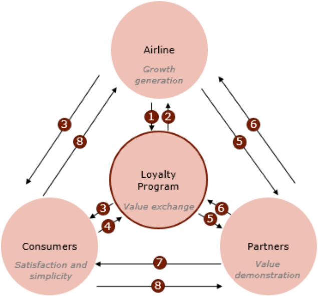
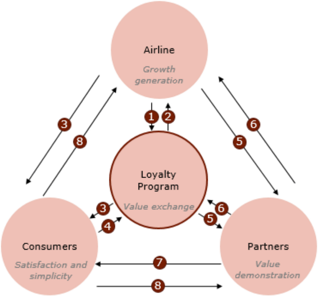


Aeroplan Loyalty Program

Accumulation

- Use Aeroplan-linked credit cards.
- Buy Air Canada (and other) flight tickets.



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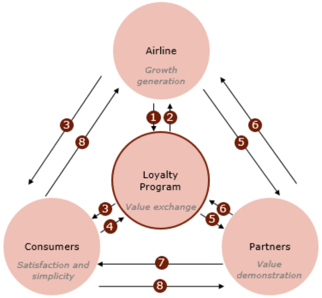

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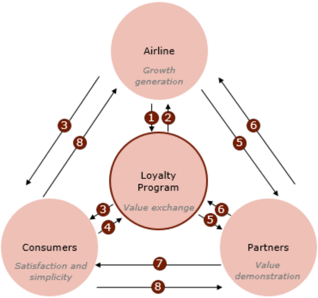

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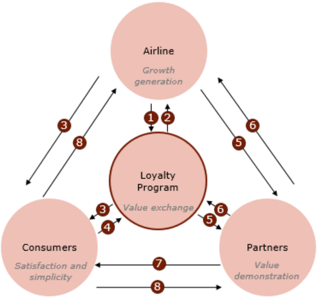
- Use Aeroplan-linked credit cards.
- Buy Air Canada (and other) flight tickets.

1. *Members* collect points from *partners*.
2. *Partners* buy points from Aeroplan.

Redemption

- Buy Air Canada (and other) flight tickets.
- Buy toasters etc.

1. Members use miles to get free tickets or toasters from Aeroplan.
2. Aeroplan pays the partner for the tickets and toasters.



Aeroplan's problem

$$\max_{\substack{\text{Miles per toaster} \\ \text{from member,} \\ \text{Budget, promotion,} \\ \text{etc.}}} \quad : \quad \sum_{p,j} (\text{Revenue} - \text{Costs})$$

Subject to

- Some meaningful constraints

Aeroplan's problem

$$\max_{\substack{\text{Miles per toaster}\\\text{from member,}\\\text{Budget, promotion,}\\\text{etc.}}} \; : \; \sum_{p,j} (\text{Accumulated miles} \times P \to AE \text{ Price} - \text{Costs})$$

Subject to

- Some meaningful constraints

Aeroplan's problem

$$\max_{\substack{\text{Miles per toaster} \\ \text{from member,} \\ \text{Budget, promotion,} \\ \text{etc.}}} : \quad \sum_{p,j} \left(d_{p,j}^{acc} \times \pi_{p,j}^{P \to AE} - \text{Costs} \right)$$

Subject to

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Aeroplan's problem

$$\max_{\substack{\text{Miles per toaster}\\\text{from member,}\\\text{Budget, promotion,}\\\text{etc.}}} \; : \; \sum_{p,j} \left(d_{p,j}^{acc} \times \pi_{p,j}^{P \to AE} - \text{Redeem miles} \times \frac{AE \to P \text{ price per toaster}}{\text{miles per toaster}} \right)$$

Subject to

- Some meaningful constraints

Aeroplan's problem

$$\max_{\substack{\text{Miles per toaster} \\ \text{from member,} \\ \text{Budget, promotion,} \\ \text{etc.}}} \quad : \quad \sum_{p,j} \left(d^{acc}_{p,j} \times \pi^{P \to AE}_{p,j} - d^{red}_{p,j} \frac{\pi^{AE \to P}_{p,j}}{m^{red}_{p,j}} \right)$$

Subject to

- Some meaningful constraints

Aeroplan's problem

$$\max_{\substack{\text{Miles per toaster}\\ \text{from member,}\\ \text{Budget, promotion,}\\ \text{etc.}}} : \sum_{p,j} \left(d^{acc}_{p,j} \times \pi^{P \to AE}_{p,j} - d^{red}_{p,j} \frac{\pi^{AE \to P}_{p,j}}{m^{red}_{p,j}} \right)$$

$$- \text{Promotion budget}$$

Subject to

- Some meaningful constraints

Aeroplan's problem

$$\max_{\substack{\text{Miles per toaster}\\\text{from member,}\\\text{Budget, promotion,}\\\text{etc.}}} \quad : \quad \sum_{p,j} \left(d^{acc}_{p,j} \times \pi^{P\to AE}_{p,j} - d^{red}_{p,j} \frac{\pi^{AE\to P}_{p,j}}{m^{red}_{p,j}} \right)$$

$$- \text{Promotion budget} - \text{Budget to get new partners/members}$$

Subject to

- Some meaningful constraints

Aeroplan's problem

$$\max_{\substack{\text{Miles per toaster} \\ \text{from member,} \\ \text{Budget, promotion,} \\ \text{etc.}}} : \sum_{p,j} \left(d^{acc}_{p,j} \times \pi^{P \to AE}_{p,j} - d^{red}_{p,j} \frac{\pi^{AE \to P}_{p,j}}{m^{red}_{p,j}} \right)$$

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Subject to

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$$\max_{\substack{\text{Miles per toaster}\\ \text{from member,}\\ \text{Budget, promotion,}\\ \text{etc.}}} \quad : \quad \sum_{p,j} \left(d^{acc}_{p,j} \times \pi^{P \to AE}_{p,j} - d^{red}_{p,j} \frac{\pi^{AE \to P}_{p,j}}{m^{red}_{p,j}} \right)$$

$$- \text{Promotion budget} - \text{Budget to get new partners/members}$$

Subject to

- Budget balance constraints.
- Miles Accumulated in a year $\geq$ Miles redeemed in a year

Aeroplan's problem

$$\max_{\substack{\text{Miles per toaster} \\ \text{from member,} \\ \text{Budget, promotion,} \\ \text{etc.}}} : \quad \sum_{p,j} \left(d_{p,j}^{acc} \times \pi_{p,j}^{P \to AE} - d_{p,j}^{red} \frac{\pi_{p,j}^{AE \to P}}{m_{p,j}^{red}} \right)$$

$$- \text{Promotion budget} - \text{Budget to get new partners/members}$$

Subject to

- Budget balance constraints.
- Miles Accumulated in a year $\geq$ Miles redeemed in a year
- Contract based promotions fulfilled.

$$
\begin{aligned}
\max_{m_{p,j}^{red}, B_{AE}, a_{p,j}, \Psi} \lambda \Bigg(& \sum_k \Bigg(\sum_{p \in \{FI,O\}} \sum_{j \in J_p} d_{M_k,p,j}^{acc} \pi_{p,j}^{P \to AE} - \sum_{p \in \{AC,FI,O\}} \sum_{j \in J_p^{red}} d_{M_k,p,j}^{red} \frac{\pi_{p,j}^{AE \to P}}{m_{p,j}^{red}} \Bigg) \\
& - B_{AE} + f(\Psi) + \sum_k \sum_{j \in J_{AC}} (p_{j,k}^{acc} s_{j,k}^{acc} + p_{j,k}^{red} s_{j,k}^{red}) - B_{AC} \Bigg) \\
& + (1-\lambda) \Bigg(\sum_k \sum_{p \in \{AC,FI,O\}} \sum_{j \in J_p} g(d_{M_k,p,j}^{acc}) \Bigg) \qquad \text{(1a)}
\end{aligned}
$$

subject to

$$
\begin{aligned}
\sum_{p \in \{AC,FI,O\}} \sum_{j \in J_p} a_{p,j} + \Psi &\leq B_{AE} && \text{(1b)} \\
\sum_k \sum_{p \in \{AC,FI,O\}} \sum_{j \in J_p} d_{M_k,p,j}^{red} &\leq \sum_k \sum_{p \in \{AC,FI,O\}} \sum_{j \in J_p} d_{M_k,p,j}^{acc} && \text{(1c)} \\
a_{p,j} &\geq l_{p,j} \quad \forall p, j && \text{(1d)}
\end{aligned}
$$

Partners' problem

$$\max_{\substack{\text{Miles acc} \\ \text{per toaster,} \\ \text{Budget, promotion,} \\ \text{etc.}}} : \sum \text{Addnl profit due to partnership} - \text{Cost of partnership}$$

Subject to

- Some meaningful constraints

Partners' problem

$$\max_{\substack{\text{Miles acc} \\ \text{per toaster,} \\ \text{Budget, promotion,} \\ \text{etc.}}} \; : \; \sum \text{Accum profit} + \text{Redem profit} - \text{Cost of partnership}$$

Subject to

- Some meaningful constraints

Partners' problem

$$\max_{\substack{\text{Miles acc}\\ \text{per toaster,}\\ \text{Budget, promotion,}\\ \text{etc.}}} \; : \; \sum p_j^{acc} s_j^{acc} + p_j^{red} s_j^{red} - \text{Cost of partnership}$$

Subject to

- Some meaningful constraints

Partners' problem

$$\max_{\substack{\text{Miles acc}\\ \text{per toaster,}\\ \text{Budget, promotion,}\\ \text{etc.}}} : \sum p_j^{acc} s_j^{acc} + p_j^{red} s_j^{red} - (\text{Accum miles} \times P \to AE \text{ Price} + B)$$

Subject to

- Some meaningful constraints

Partners' problem

$$\max_{\substack{\text{Miles acc} \\ \text{per toaster,} \\ \text{Budget, promotion,} \\ \text{etc.}}} \quad : \quad \sum p_j^{acc} s_j^{acc} + p_j^{red} s_j^{red} - (d_{p,j}^{acc} \times \pi_{p,j}^{P \rightarrow AE} + B)$$

Subject to

- Some meaningful constraints

Partners' problem

$$\max_{\substack{\text{Miles acc}\\\text{per toaster,}\\\text{Budget, promotion,}\\\text{etc.}}} \quad : \quad \sum p_j^{acc} s_j^{acc} + p_j^{red} s_j^{red} - (d_{p,j}^{acc} \times \pi_{p,j}^{P \to AE} + B)$$

Subject to

- $d_{p,j}^{acc} = s_j^{acc} m_{p,j}^{acc}$
- $d_{p,j}^{red} = s_j^{red} m_{p,j}^{red}$

Partners' problem

$$\max_{\substack{\text{Miles acc}\\ \text{per toaster,}\\ \text{Budget, promotion,}\\ \text{etc.}}} \quad : \quad \sum p_j^{acc} s_j^{acc} + p_j^{red} s_j^{red} - (d_{p,j}^{acc} \times \overset{P \to AE}{\pi_{p,j}} + B)$$

Subject to

- $d_{p,j}^{acc} = s_j^{acc} m_{p,j}^{acc}$
- $d_{p,j}^{red} = s_j^{red} m_{p,j}^{red}$
- Budget balance constraint.

Partners' problem

$$\max_{\substack{\text{Miles acc} \\ \text{per toaster,} \\ \text{Budget, promotion,} \\ \text{etc.}}} : \sum p_j^{acc} s_j^{acc} + p_j^{red} s_j^{red} - (d_{p,j}^{acc} \times \overset{P \to AE}{\pi_{p,j}} + B)$$

Subject to

- $d_{p,j}^{acc} = s_j^{acc} m_{p,j}^{acc}$
- $d_{p,j}^{red} = s_j^{red} m_{p,j}^{red}$
- Budget balance constraint.
- Stock availability constraint

Partners' problem

$$\max_{\substack{\text{Miles acc} \\ \text{per toaster,} \\ \text{Budget, promotion,} \\ \text{etc.}}} \quad : \quad \sum p_j^{acc} s_j^{acc} + p_j^{red} s_j^{red} - (d_{p,j}^{acc} \times \overset{P \to AE}{\pi_{p,j}} + B)$$

Subject to

- $d_{p,j}^{acc} = s_j^{acc} m_{p,j}^{acc}$
- $d_{p,j}^{red} = s_j^{red} m_{p,j}^{red}$
- Budget balance constraint.
- Stock availability constraint $s_j^{acc} + s_j^{red} \leq \alpha_j$ and $s_j^{red} \leq \beta_j$.

Partners' problem

$$\max_{\substack{\text{Miles acc}\\ \text{per toaster,}\\ \text{Budget, promotion,}\\ \text{etc.}}} \quad : \quad \sum p_j^{acc} s_j^{acc} + p_j^{red} s_j^{red} - (d_{p,j}^{acc} \times \pi_{p,j}^{P \to AE} + B)$$

Subject to

- $d_{p,j}^{acc} = s_j^{acc} m_{p,j}^{acc}$
- $d_{p,j}^{red} = s_j^{red} m_{p,j}^{red}$
- Budget balance constraint.
- Stock availability constraint $s_j^{acc} + s_j^{red} \leq \alpha_j$ and $s_j^{red} \leq \beta_j$.

Note: Air Canada's problem can be concatenated with Aeroplan's.

$$\max_{m^{acc}_{p,j}, B_p, b_{p,j}} \quad \sum_k \left(\sum_{j \in J_p} \left(p^{acc}_{j,k} s^{acc}_{j,k} - d^{acc}_{M_k,p,j} \overset{P \to AE}{\pi}_{p,j} \right) + \sum_{j \in J^{red}_p} p^{red}_{j,k} s^{red}_{j,k} \right) - B_p \tag{2a}$$

subject to

$$\sum_{j \in J_p} b_{p,j} \leq B_p \tag{2b}$$

$$d^{acc}_{M_k,p,j} = s^{acc}_{j,k} m^{acc}_{p,j} \tag{2c}$$

$$d^{red}_{M_k,p,j} = s^{red}_{j,k} m^{red}_{p,j} \tag{2d}$$

$$\sum_k s^{acc}_{j,k} + s^{red}_{j,k} \leq \alpha_j \tag{2e}$$

$$\sum_k s^{red}_{j,k} \leq \beta_j \tag{2f}$$

A regression model

$$s^{acc}_{k,p,j,t} = \mathcal{R}\left(\{m^{red}_{p,j,t-1}\}_{p,j}, m^{acc}_{p,j,t}, a_{p,j}, b_{p,j}\right) \quad (3a)$$

$$s^{red}_{k,p,j,t} = \mathcal{R}\left(\{m^{red}_{p,j,t}\}_{p,j}, s^{acc}_{k,p,j,t-1}, m^{acc}_{p,j,t-1}, a_{p,j}, b_{p,j}\right) \quad (3b)$$

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s^{red}_{k,p,j,t} &= \mathcal{R}\left(\{m^{red}_{p,j,t}\}_{p,j}, s^{acc}_{k,p,j,t-1}, m^{acc}_{p,j,t-1}, a_{p,j}, b_{p,j}\right) & \text{(3b)}
\end{aligned}
$$

One predicts the number of each product bought for the sake of accumulating miles or for redeeming miles using a regression model (perhaps a neural network), given

A regression model

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One predicts the number of each product bought for the sake of accumulating miles or for redeeming miles using a regression model (perhaps a neural network), given

- The miles accumulated in previous time period
- The miles redeemed in previous time period

A regression model

$$
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One predicts the number of each product bought for the sake of accumulating miles or for redeeming miles using a regression model (perhaps a neural network), given

- The miles accumulated in previous time period
- The miles redeemed in previous time period
- Promotion efforts by Aeroplan
- Promotion efforts by the partner

A regression model

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s^{acc}_{k,p,j,t} &= \mathcal{R}\left(\{m^{red}_{p,j,t-1}\}_{p,j}, m^{acc}_{p,j,t}, a_{p,j}, b_{p,j}\right) && \text{(3a)}\\
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- Promotion efforts by Aeroplan
- Promotion efforts by the partner
- Perhaps member type? Literacy in the program, income level?

A regression model

$$s^{acc}_{k,p,j,t} = \mathcal{R}\left(\{m^{red}_{p,j,t-1}\}_{p,j}, m^{acc}_{p,j,t}, a_{p,j}, b_{p,j}\right) \tag{3a}$$

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One predicts the number of each product bought for the sake of accumulating miles or for redeeming miles using a regression model (perhaps a neural network), given

- The miles accumulated in previous time period
- The miles redeemed in previous time period
- Promotion efforts by Aeroplan
- Promotion efforts by the partner
- Perhaps member type? Literacy in the program, income level?
- Other interesting parameters?

Question.

Why is this problem hard?

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  - Partner's decision of $m_{p,j}^{acc}$ affects Aeroplan's revenue.

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- We need a solution that *simultaneously* optimizes all players' optimization problems.

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Welcome to Game theory

Example: Prisoner's dilemma

The prisoners simultaneously select their strategy.

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The prisoners simultaneously select their strategy.

		Prisoner B			
		silent		betray	
Prisoner A	silence	-1	-1	-3	0
	betray	0	-3	-2	-2

Example: Prisoner's dilemma

The prisoners simultaneously select their strategy.

There is a unique Nash equilibrium:

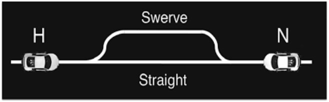
(*betray*, *betray*)

		Prisoner B			
		silent		betray	
Prisoner A	silence	-1	-1	-3	0
	betray	0	-3	-2	-2

Example: game of chicken

Simultaneously, the cars decide to swerve and straight.

		Car B			
		Swerve		Straight	
Car A	Swerve	tie 0	tie 0	lose -1	win +1
	Straight	win +1	lose -1	accident -1000	accident -1000



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Example: game of chicken

Simultaneously, the cars decide to swerve and straight.

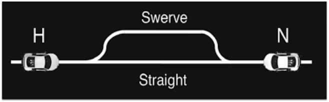
There are 3 equilibria:

Car A: swerve and Car B: straight

Car A: straight and Car B: swerve

Car A: swerve with prob $\frac{999}{1000}$ and straight wih prob $\frac{1}{1000}$
Car B: swerve with prob $\frac{999}{1000}$ and straight with prob $\frac{1}{1000}$

		Car B			
		Swerve		Straight	
Car A	Swerve	tie 0	tie 0	lose -1	win +1
	Straight	win +1	lose -1	accident -1000	accident -1000



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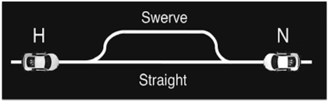

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Example: game of chicken

Simultaneously, the cars decide to swerve and straight.

There is a correlated equilibrium where:

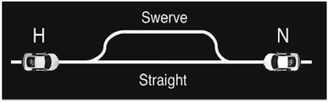
Car A and Car B get a positive utility!

Car A plays *swerve* and Car B plays *swerve* with prob $\frac{998}{1000}$,

Car A plays *swerve* and Car B plays *straight* with prob $\frac{997}{1000^2}$

Car A plays *straight* and Car B plays *swerve* with prob $\frac{999}{1000^2}$

		Car B			
		Swerve		Straight	
Car A	Swerve	tie 0	tie 0	lose -1	win +1
	Straight	win +1	lose -1	accident -1000	accident -1000



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Example: game of chicken

Simultaneously, the cars decide to swerve and straight.

Moral: by sending signals players can find a more balanced solutions.

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Car A plays *swerve* and Car B plays *straight* with prob $\frac{997}{1000^2}$

Car A plays *straight* and Car B plays *swerve* with prob $\frac{999}{1000^2}$

		Car B			
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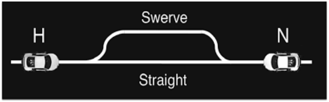
Car A	Swerve	tie 0	tie 0	lose -1	win +1
	Straight	win +1	lose -1	accident -1000	accident -1000



origem: www.researchgate.net/publication/261351299_The_effects_of_neuromodulation_on_human-robot_interaction_in_games_of_conflict_and_cooperation/figures?lo=1&utm_source=google&utm_medium=organic

Normal form games can be solved *reasonably fast* to obtain a *mixed-strategy Nash or correlated equilibrium*.

Players have a finite number of strategies and the game is represented through a multidimensional matrix with an entry for each pure profile of strategies.

		Player 2		
		rock	scissors	paper
	rock	(0,0)	(1,-1)	(-1,1)
Player 1	scissors	(-1,1)	(0,0)	(1,-1)
	paper	(1,-1)	(-1,1)	(0,0)

Table: Rock-scissors-paper game

Algorithmic approach: overview

- Step 1 Compute an initial set of strategies for each player.
- Step 2 Obtain the normal-form game associated with the enumerated strategies.
- Step 3 Compute the equilibria of the current normal-form game.
- Step 4 Determine whether there is a player with incentive to deviate. If the deviation incentive is greater than a certain tolerance, update the normal-form game with new strategies and go back to **Step 3**. Else, **return** the current equilibrium.

Step 1: Initial set of strategies

- Current strategies of the players.
- Strategies that guarantee a minimum profit for each player.
- Optimal strategies if each player controlled the remaining ones.
- Equilibrium strategies for subsets of players.
- etc...

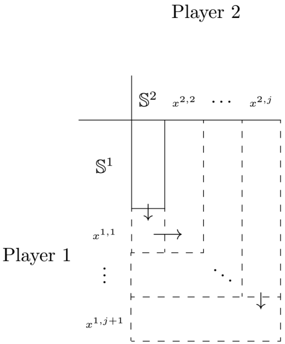
Depending on this initialization the algorithm convergence rate will vary.

Let $\mathbb{S}_p$ be the initial set of pure strategies for player p.

Step 2: Normal-form Game

Compute the utility of each player for any combination of strategies and build the associated multidimensional matrix.

Step 3: Compute the Equilibria



Nash equilibria: there are solvers available.
Correlated equilibria: there is a solver for 2-player game.

Step 4: Verify Nash Equilibria

For an equilibrium x^*, for each player p solve

$$\hat{x}_p = argmax_{x_p \in X_p} f_p(x_p, x^*_{-p})$$

If $f_p(\hat{x}_p, x^*_{-p}) - f(x^*) \leq \epsilon \quad \forall p$ then return x^*.
Otherwise, add $\hat{x}_p$ to the normal-form game.

Step 4: Verify Correlated Equilibria

For an equilibrium σ^* (probability distribution over any outcome of the normal-form game), for each player p and for each $x_p \in \mathbb{S}_p$ (strategies in the current normal-form game) solve

$$z_p = \min_{\hat{x}_p \in X_p} \sum_{x_{-p} \in \mathbb{S}_{-p}} \sigma^*(x_p, x_{-p}) f_p(x_p, x_{-p}) - \sum_{x_{-p} \in \mathbb{S}_{-p}} \sigma^*(x_p, x_{-p}) f_p(\hat{x}_p, x_{-p})$$

If z_p is negative then σ^* is not a correlated equilibrium and $\hat{x}_p$ must be added to the game.
Otherwise, if $z_p \geq 0 \quad \forall p \forall x_p \in \mathbb{S}_p$, return σ^*.

Probability based Strategy

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Probably-based strategy profile can be hard to implement.

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- If the game is only played a few times, maximizing *expected* returns does not make sense.
- A deterministic equilibrium might fail to exist! (Example: Rock-Paper-Scissors).
- Are there intelligent hacks to implementing the equilibrium?

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- The grand coalition between Aeroplan and *all* the partners creates a total increased utility.
- How can the dollar value of the utility be distributed among the partners and Aeroplan - in a fair and acceptable way?

Notions of cooperative game theory

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- These notions give indications to Aeroplan on who has the market power and where the fair rates are for each partner.

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  - Ideas from Vielma et. al. (2018) [arxiv:1811.01988].
- Interesting hacks to implement a probability-based equilibrium.

Thank you

Solution Concepts

Definition (Correlated Equilibrium. Aumann 1974)

Let $\Gamma = \langle I, (A_i)_{i \in I}, (u_i)_{i \in I} \rangle$ be a strategic form game. A *Correlated Equilibrium* is a distribution $\mu \in \Delta(A)$ such that

$$\forall i \in I,\ \forall a_i, a_i' \in A_i,\ \sum_{a_{-i} \in A_{-i}} \mu(a_i, a_{-i})[u_i(a_i, a_{-i}) - u_i(a_i', a_{-i})] \geq 0.$$

Figure: An example of a Correlated Equilibrium.

Nash Equilibrium

Definition (Nash Equilibrium. Nash, 1950)

Let $\Gamma = \langle I, (A_i)_{i\in I}, (u_i)_{i\in I} \rangle$ be a strategic form game. A mixed strategy of player i is a probability measure over A_i, we denote the set of mixed strategy profiles of player i as Δ_i. A mixed strategy profile $\delta^* = (\delta_i^*)_{i\in I}$ is a mixed Nash Equilibrium iff

$$\forall i \in I,\ \forall \delta_i \in \Delta_i,\ u_i(\delta_i^*, \delta_{-i}) - u_i(\delta_i, \delta_{-i}) \geq 0.$$

Remark A mixed Nash Equilibrium is a Correlated Equilibrium where the players' mixed strategies are independent.

Cooperative Game Theory - Core

Definition (Core)

For a Cooperative Game (N, v), a payoff distribution $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n$ is

- *efficient* if $\sum_{i \in N} x_i = v(N)$,
- *individually rational* if for every $i \in N$, $x_i \geq v(\{i\})$,
- *coalitionally rational* if for every $B \in 2^N \setminus \emptyset$, $\sum_{i \in B} x_i \geq v(\{B\})$.

The *Core* of (N, v) is the set $C(N, v) = \{\mathbf{x} \in \mathbb{R}^n \,|\, \mathbf{x} \text{ is efficient and coalitionally rational}\}$

Remark The Core could be large, small or even empty.

Shapley value

Definition (Shapley value)

The Shapley value of a cooperative game (N, v) is a payoff vector $\Phi(N, v)$ whose i-th coordinate is
$\Phi_i(N, v) = \sum_{B \subseteq N : i \notin B} \frac{|B|!(n-|B|-1)!}{n!}[v(B \cup \{i\}) - v(B)]$.

Remark The Shapley value assigns to each player her average marginal contribution to the game.
Note that the Shapley value might not be in the Core.