# The Friction Stir Welding Process

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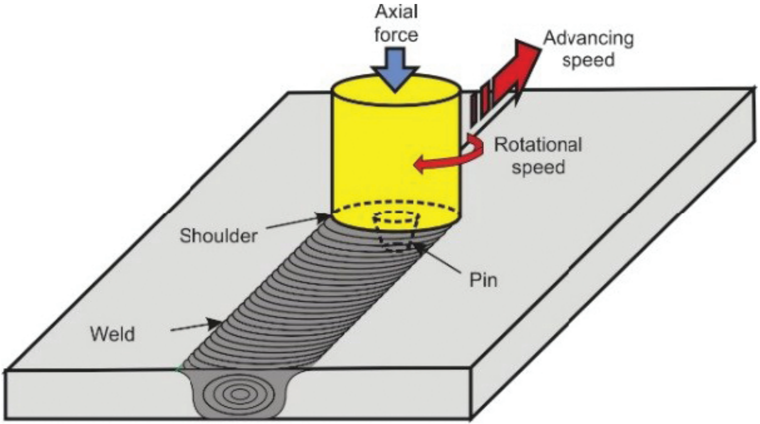
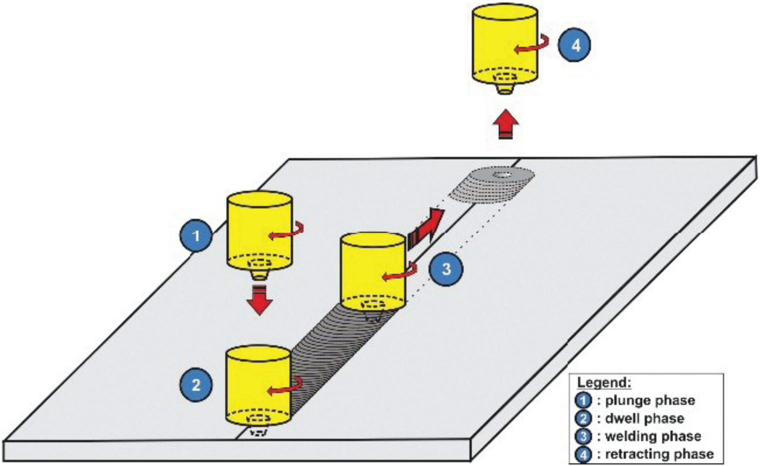
# The FSW process



# The Four Phases of Creating a Weld



# Mechanisms of Welding

Steady State:

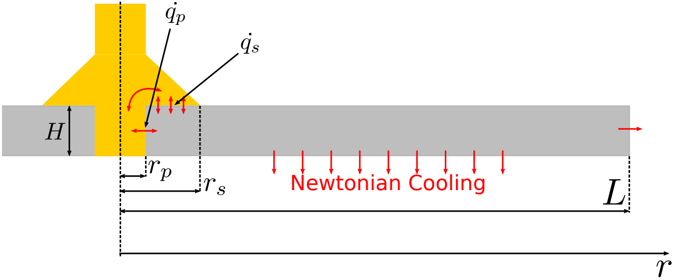
- Heat
- Shear flow
- Applied pressure
- Plastic/elastic

Parameters:

| | Aluminium | Steel | |
|---|---|---|---|
| $k$ | 237 | 24 | $\mathrm{J/s.m.K}$ |
| $E$ | 70 | 210 | $\mathrm{GPa}$ |
| $\nu$ | 0.35 | 0.3 | |
| $\rho$ | 2700 | 8170 | $\mathrm{kg/m^3}$ |
| $c_p$ | 897 | 418 | $\mathrm{J/kg.K}$ |

# What Role Does Heat Play?

# The heat model

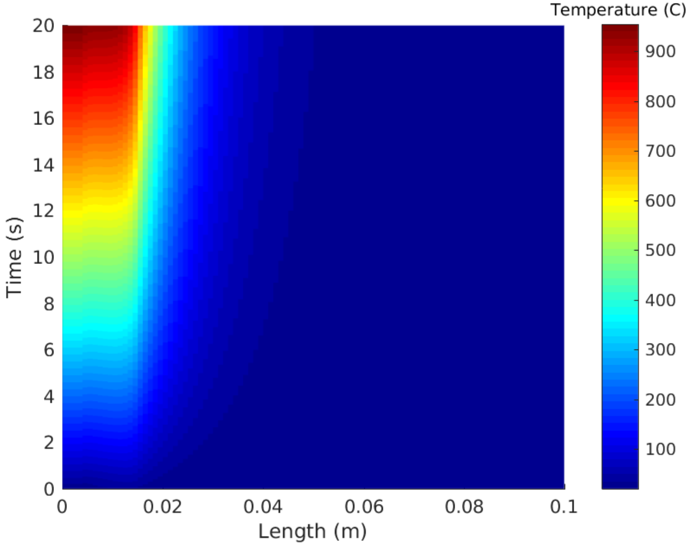



$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[ r k \frac{\partial T}{\partial r} \right] + \dot{q}_s \Big|_{(r_p, r_s]}$$

$$\rho_p c_p \frac{\mathrm{d}T_p}{\mathrm{d}t} = \dot{q}_p \Big|_{r_p} + \frac{h_{ps}}{V_p}(T_p - T_s) + \frac{h_p}{V_p}(T_p - T|_{r_p})$$

$$\rho_s c_s \frac{\mathrm{d}T_s}{\mathrm{d}t} = -\frac{h_{ps}}{V_s}(T_p - T_s) + \frac{h_s}{V_s}(T_s - T|_{(r_p, r_s]})$$

# The Heat Equation

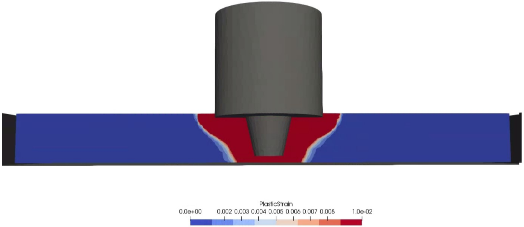


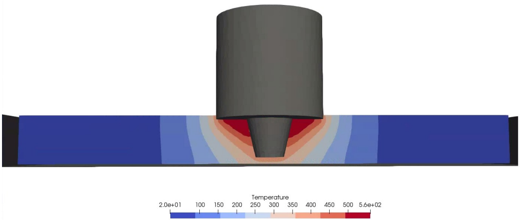
# Lessons Learned

1. The shoulder is mostly responsible for heating, not the pin.
2. The heat loss through the tool is significant.
3. The heat does not travel too far in the $r$ direction, due to the large heat capacity of the material.
4. The effect of yielding material should be taken into consideration to avoid the constant rise in temperature. The material should yield in less than 10 s.

# Plastic / Elastic

# Elastic/Plastic



# Elastic/Plastic Equations

Elastic equations

$$
\begin{aligned}
\nabla \cdot \sigma &= 0 && (1)\\
\sigma &= \mathbb{C}\varepsilon && (2)\\
\varepsilon &= \frac{1}{2}(\nabla \vec{u} + \nabla \vec{u}^T) && (3)
\end{aligned}
$$

Heat Equation

$$
\rho c \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + \sigma : \varepsilon \qquad (4)
$$

Plastic equations

$$
\nabla \cdot \sigma = \rho \frac{D\vec{v}}{Dt} \qquad \nabla \cdot \vec{v} = 0 \qquad (5)
$$

$$
\dot{\varepsilon} = \frac{\Lambda}{\sigma_y}(\sigma - \frac{1}{3}\mathrm{Tr}(\sigma)\mathbb{I}) = \frac{\Lambda}{\sigma_y}\sigma^{dev} \qquad (6)
$$

$$
\sigma_y^2(T) = \frac{3}{2}\sigma_{ij}^{dev}\sigma_{ij}^{dev} \qquad (7)
$$

# Plastic I ($r_p \le r \le r_b$)

- Force balance: $\vec{v} = \langle v_r(r), v_\theta(r), 0 \rangle$

$$
\begin{aligned}
\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r}\right) &= \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\left(\sigma_{rr} - \sigma_{\theta\theta}\right), \quad (8)\\
\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_r v_\theta}{r}\right) &= \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{2}{r}\sigma_{r\theta}, \quad (9)\\
\frac{\partial v_r}{\partial r} + \frac{v_r}{r} &= 0. \quad (10)
\end{aligned}
$$

- Constitutive relation:

$$
\sigma_{rr} = p + \frac{\sigma_y}{\Lambda}\frac{\partial v_r}{\partial r}, \qquad \sigma_{\theta\theta} = p + \frac{\sigma_y}{\Lambda}\frac{v_r}{r}, \quad (11)
$$

$$
\sigma_{r\theta} = \frac{\sigma_y}{2\Lambda}\left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right), \qquad p = \frac{1}{3}\left(\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz}\right), \quad (12)
$$

$$
(\sigma_{rr} - p)^2 + (\sigma_{\theta\theta} - p)^2 + (\sigma_{zz} - p)^2 + 2\sigma_{r\theta}^2 = \frac{2\sigma_y^2}{3}. \quad (13)
$$

# Elastic I ($r_b \le r \le L$)

- Force balance: $\vec{u} = \langle u_r(r), u_\theta(r), 0 \rangle$

$$0 = \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\left(\sigma_{rr} - \sigma_{\theta\theta}\right), \tag{14}$$

$$0 = \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{2}{r}\sigma_{r\theta}. \tag{15}$$

- Constitutive relation:

$$\sigma_{rr} = (\lambda + 2\mu)\varepsilon_{rr} + \lambda\varepsilon_{\theta\theta}, \quad \sigma_{r\theta} = 2\mu\varepsilon_{r\theta}, \tag{16}$$

$$\sigma_{\theta\theta} = (\lambda + 2\mu)\varepsilon_{\theta\theta} + \lambda\varepsilon_{rr}, \quad \sigma_{zz} = \lambda(\varepsilon_{\theta\theta} + \varepsilon_{rr}), \tag{17}$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{r\theta} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right), \tag{18}$$

$$\varepsilon_{\theta\theta} = \frac{u_r}{r}. \tag{19}$$

# Boundary conditions

- $r = r_p$:

$$v_\theta = \gamma \omega r_p, \tag{20}$$
$$\sigma_{r\theta} = f \sigma_{rr}. \tag{21}$$

- $r = L$:

$$\sigma_{rr} = 0, \tag{22}$$
$$\sigma_{r\theta} = 0. \tag{23}$$

- $r = r_b$:

$$v_\theta = 0, \tag{24}$$
$$p = p_b, \tag{25}$$
$$[\sigma_{rr}] = 0, \tag{26}$$
$$[\sigma_{r\theta}] = 0. \tag{27}$$

# Solution for the elastic deformation in steady state

$$\sigma_{rr} = \sigma^b_{rr}\frac{r_b^2}{r^2}, \quad \sigma_{\theta\theta} = -\sigma^b_{rr}\frac{r_b^2}{r^2}, \quad \sigma_{r\theta} = \sigma^b_{r\theta}\frac{r_b^2}{r^2}, \tag{28}$$

$$u_r = -\sigma^b_{rr}\frac{1+\nu}{E}\frac{r_b^2}{r}, \quad u_\theta = -\sigma^b_{r\theta}\frac{1+\nu}{E}\frac{r_b^2}{r} \tag{29}$$

## Solution for the plastic deformation in steady state

Incompressibility gives $v_r = \dfrac{c_1}{r}$, and from the flow relations

$$\Lambda = \left( \frac{3\sigma_y^2 c_1^2}{r^4 \left( \sigma_y^2 - 3\sigma_{r\theta}^2 \right)} \right)^{\frac{1}{2}}.$$

Shear force balance gives

$$\sigma_{r\theta} = \frac{\rho c_1 v_\theta}{r} + \frac{c_2}{r^2},$$

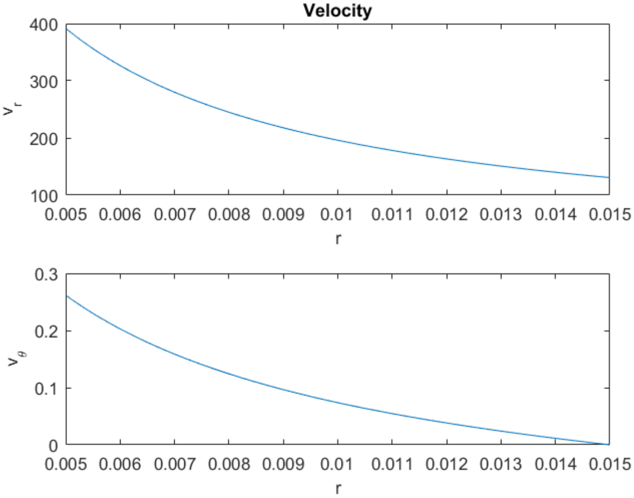
and the remaining flow relation gives

$$\frac{\partial v_\theta}{\partial r} = \left( \frac{1}{r} + \frac{2\Lambda \rho c_1}{\sigma_y r} \right) v_\theta + \frac{2\Lambda c_2}{\sigma_y r^2}, \quad v_\theta = \begin{cases} \gamma \omega r_p & \text{at } r_p \\ 0 & \text{at } r_b, \end{cases} \tag{30}$$

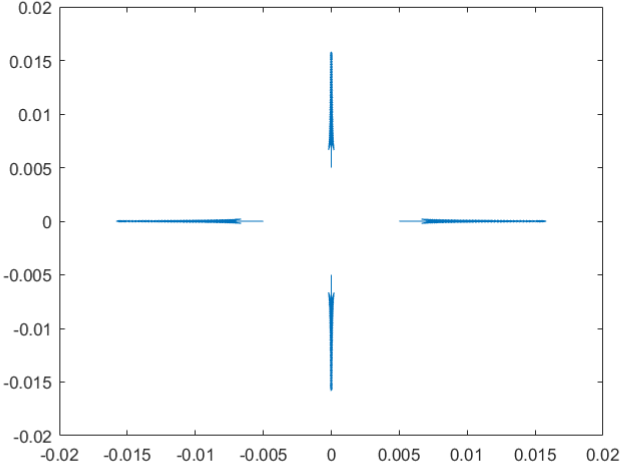
with the force balance giving

$$\frac{\partial \sigma_{rr}}{\partial r} = \frac{2\sigma_y c_1}{\Lambda r^3} - \frac{\rho c_1^2}{r^3} - \frac{\rho v_\theta^2}{r}, \qquad \begin{cases} \sigma_{r\theta} = f\sigma_{rr} & \text{at } r_p \\ p = p_b & \text{at } r_b. \end{cases} \tag{31}$$

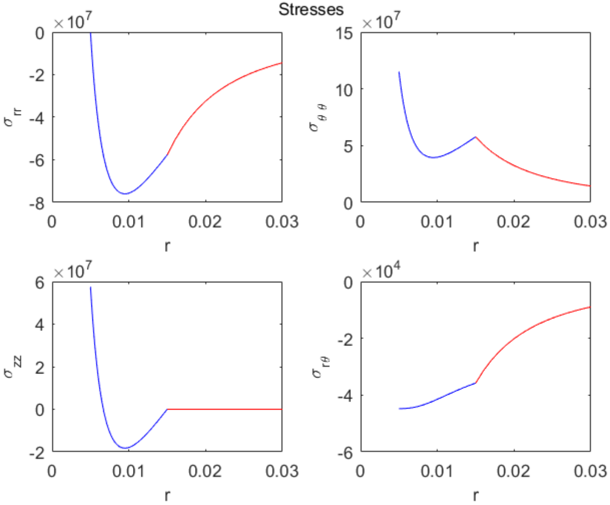
# Solution for the plastic deformation in steady state



# Solution for the plastic deformation in steady state



# Solution for the elastic/plastic deformation



# Lessons learned

- If $v_r = 0$ (stationary at pin) solution does not exist
- If $v_r > 0$ (unphysical) then $v_r$ is huge
- Inconsistency of plastic flow with incompressible fluid
- Perhaps a shear stress condition at $r_p$ is unwise

# Plastic II

Equations:

$$0 = \frac{d\sigma_{rr}}{dr} + \frac{1}{r}\left(\sigma_{rr} - \sigma_{\theta\theta}\right), \tag{32}$$

$$0 = \frac{d\sigma_{r\theta}}{dr} + \frac{2}{r}\sigma_{r\theta}, \tag{33}$$

$$\frac{1}{4}(\sigma_{rr} - \sigma_{\theta\theta})^2 + \sigma_{r\theta}^2 = \sigma_y^2(T) \tag{34}$$

Boundary conditions:

$$\sigma_{rr} = P_0, \quad \sigma_{r\theta} = f\sigma_{rr}, \quad r = r_p. \tag{35}$$

Solutions:

$$\sigma_{r\theta}(r) = fP_0 r_p^2/r^2, \tag{36}$$

$$\sigma_{rr}(r) = -\int_{r_p}^{r} \frac{2}{s}\sqrt{\sigma_y^2 - \sigma_{r\theta}^2(s)}ds + P_0, \tag{37}$$

$$\sigma_{\theta\theta} = \sigma_{rr} \pm 2\sqrt{\sigma_y^2 - \sigma_{r\theta}^2(r)}. \tag{38}$$

Use yield criterion and two matching conditions $[\sigma_{rr}] = [\sigma_{r\theta}] = 0$ to determine $r_b$ and $\sigma_{rr}^b, \sigma_{r\theta}^b$. Decouples flow and stress. Temperature could be useful: $\sigma_y = \sigma_y(r)$

# Stokes Flow + Heat Equation

Numerical simulation with finite element method using Stokes model.

$$\nabla \cdot (2\mu\dot{\varepsilon}) - \nabla p = 0, \tag{39}$$

$$\nabla \cdot v = 0, \tag{40}$$

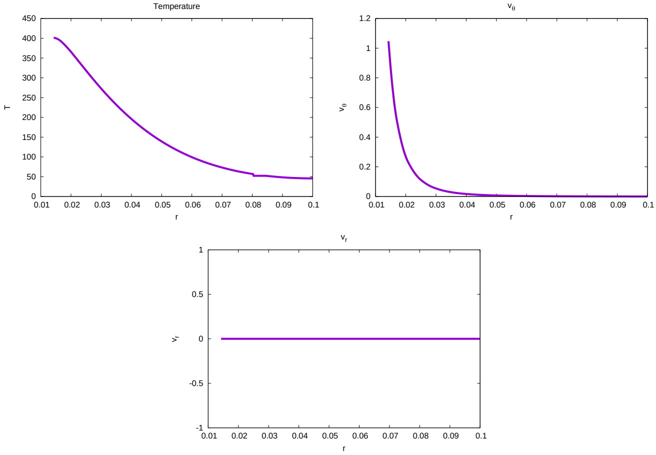
$$\rho C \frac{DT}{Dt} - \nabla \cdot (k \nabla T) - \dot{Q} = 0 \tag{41}$$

$$\dot{Q} = \alpha_{dissipation} \sigma^{dev} : \dot{\varepsilon} \text{ with } \sigma^{dev} = 2\mu\dot{\varepsilon} \tag{42}$$

$$\dot{\varepsilon} = \frac{1}{2}(\nabla^T(v) + \nabla(v)) \tag{43}$$

Norton-Hoff law: $\mu = K(T)(2\dot{\varepsilon} : \dot{\varepsilon} + 3\gamma^2)^{\frac{m(T)-1}{2}}$.
We assume K and m constant: K = $10^6$ and $m = 0.2$ , $\gamma = 10^{-3}$.

# Numerical simulation with finite element method




Stress components seem unstable. Boundary layer?
Temperature is uncoupled.
Still need $K = K(T),\ m = m(T)$.

# Numerical simulation for plastic deformation FV

We derive the following system with the unkowns $(v_r, v_\theta, \Lambda, p)$

$$\rho\frac{\partial v_r}{\partial t} + (\rho v_r - \frac{\sigma_y}{\Lambda r})\frac{\partial v_r}{\partial r} - \frac{\partial}{\partial r}(\frac{\sigma_y}{\Lambda}\frac{\partial v_r}{\partial r}) + \frac{\sigma_y}{\Lambda r^2}v_r = \frac{\rho}{r}v_\theta^2 + \frac{\partial p}{\partial r},$$

$$\rho\frac{\partial v_\theta}{\partial t} + (\rho v_r - \frac{\sigma_y}{\Lambda r})\frac{\partial v_\theta}{\partial r} + \frac{\partial}{\partial r}(\frac{\sigma_y}{2\Lambda r}v_\theta - \frac{\sigma_y}{2\Lambda}\frac{\partial v_\theta}{\partial r}) + (\frac{\rho v_r}{r} + \frac{\sigma_y}{\Lambda r^3})v_\theta = 0,$$

$$\Lambda = \sqrt{\frac{3}{2}\Big(2(\frac{v_r}{r})^2 + \frac{1}{2}(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r})^2\Big)},$$

and

$$-\frac{\partial}{\partial r}(r\frac{\partial p}{\partial r}) = -\frac{\partial}{\partial r}\Big(r\frac{\partial}{\partial r}(\frac{\sigma_y}{\Lambda r}v_r) + \frac{2\sigma_y}{\Lambda r}v_r + \rho r v_r\frac{\partial v_r}{\partial r} - \rho v_\theta^2\Big)$$

closed by the boundary conditions

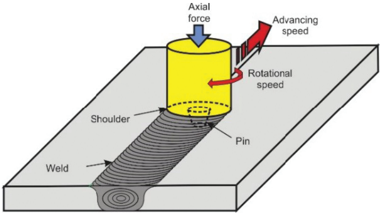
$$v_\theta = \gamma r_p \omega, \quad \sigma_{rr} = 10^5, \quad \text{at } r = r_p,$$
$$v_\theta = 0, v_r = 0, \quad \text{at } r = L.$$

# Numerical simulation for plastic deformation FV

To be done later: pressure equation is unstable.

# Proposed consistent mechanism

- Plasticize region with pressure $\sigma_{zz}$
- Heating is secondary
- $u_z$ or $\frac{\partial u_\theta}{\partial z}$ is important (may resolve incompressibility problem)
- Translation of pin within plastic region generates flow
- Shear flow blending the material



Thank you!