Regime switching in stock-bond correlations

Project submitted by
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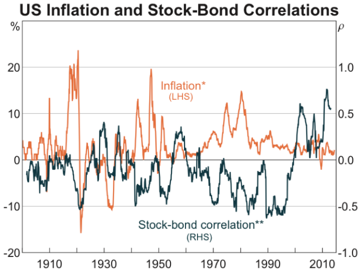
- Correlations summarize the linear relationship between two assets and is one of the building blocks of a diversified portfolio.
- The dynamics of correlation can have serious implications for the diversification and the efficiency of a portfolio.

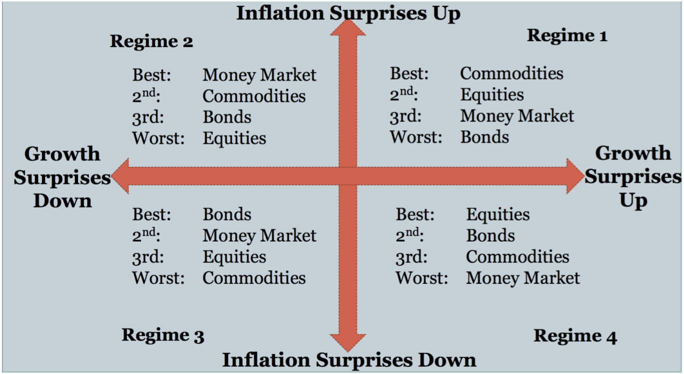


* Year-on-year change in the consumer price index; Federal Reserve Bank of New York's Index of General Prices used prior to 1913

** Three-year rolling centred correlation of monthly changes in 10-year government bond yields and S&P 500; Standard Statistics' 90 Stock Composite Index used prior to 1957

Sources: Global Financial Data; RBA

- **How to relate this dynamics to a finite number of macroeconomic factors ?**



- **Objectives** :
  1. Define regimes based on macroeconomic explanatory variables.
  2. Build a model for correlations dynamics based on transitions between macroeconomics regimes.

- Daily data of DEX bond index and S&PTSX 60 index futures from 2011 to 2016.

- Monitoring the stock and bond returns as functions of upward and downward daily movement of inflation and growth.

	1	2	3	4
μ^S	1.23	-0.02	-1.15	0.14
μ^B	-0.48	-0.43	0.48	0.34
σ^S	0.13	0.11	0.14	0.12
σ^B	0.03	0.03	0.03	0.03
ρ	-0.19	0.05	-0.30	-0.11

- The naive regimes do not describe different periods of time and can not help as predictors.

A Hidden Markov Model (HMM) approach

- The observed stock and bond returns both depend on **latent**, i.e. not directly observed, variables $\boldsymbol{Z}_t$.

- In the discrete HMM, the latent variables take on K different values (regimes). The dynamics of the latent variables is described by the transition matrix $\boldsymbol{Q}$.

- In the Gaussian model, the conditional distribution of the log returns is normal

$$\boldsymbol{R}_t \equiv \begin{pmatrix} R_t^B \\ R_t^S \end{pmatrix} \Big| \boldsymbol{Z}_t = i \sim \mathcal{N}(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i), \qquad i = 1, \cdots, K.$$

- The parameters $\boldsymbol{Q}, \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i$ can be estimated from historical data using maximum likelihood estimation or the expectation-maximisation (EM) algorithm.

- Estimation of the model with DEX bond index and S&PTSX 60 index futures from 2011 to 2016 : 5 regimes.

mu =

-0.0397	$\boxed{-0.7980}$	$\boxed{0.6491}$	0.2788	-0.2045
0.0199	0.0635	$\boxed{-0.3429}$	$\boxed{0.0926}$	0.0394

sigma =

0.1294	0.0957	0.0537	0.0664	0.2258
0.0356	0.0439	0.0476	0.0242	0.0585

rho =

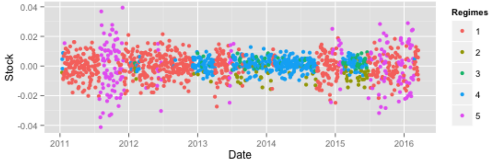
-0.3879	0.1281	0.1744	-0.1710	-0.4298

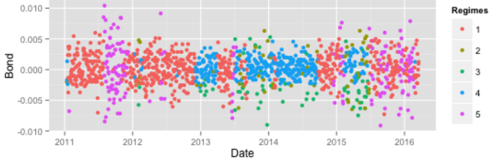
Q =

0.9815	0.0000	0.0000	0.0027	0.0158
0.0057	0.3466	0.2938	0.3384	0.0155
0.0000	0.5951	0.3226	0.0822	0.0000
0.0079	0.0020	0.1530	0.8367	0.0003
0.0342	0.0002	0.0000	0.0096	0.9560

nu =

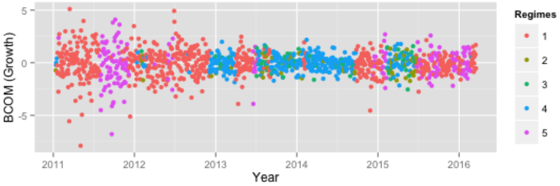
0.4623	0.0695	0.0755	0.2008	0.1919





A Hidden Markow Model (HMM) approach : Results (II)

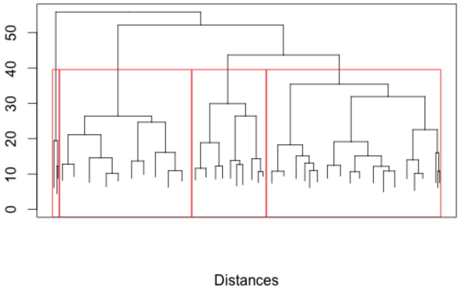
- Estimation of the model with DEX bond index and S&PTSX 60 index futures from 2011 to 2016 : 5 regimes.
- Evolution of inflation and growth proxies.



- The meaning of the regimes is **not clearly related** to macroeconomics factors.

- Clustering is a set of learning algorithms which automatically group ”similar” observations into clusters.
- Feed the learning algorithm with a number of financial variables acting as inflation and growth proxies : currency exchange rates, commodities indices, volume flows, ..., etc.
- **What can we explore with it ?**
  1. Is the number of clusters the same as the number of regimes found in HMM ?
  2. Are the stock-bond correlations distinct across the different clusters ?
  3. Are the data within each cluster normally distributed ?
- **Benefits** :
  1. Additional input from the additional variables.
  2. Better connection between the stock-bond correlations and the macroeconomic factors.

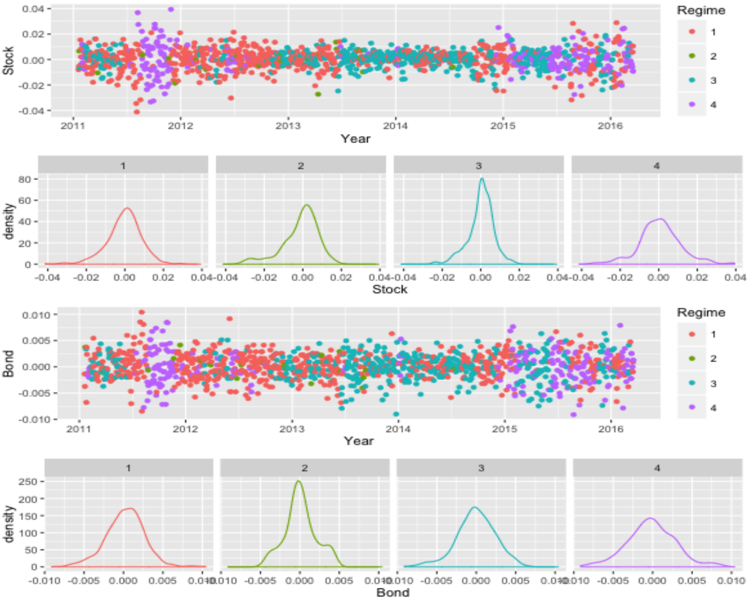
- Each observation starts in its own cluster then clusters are gradually merged according to their relative distance.
- Information about past data is needed for the algorithm to **grasp the time dimension of the data**.



	1	2	3	4
μ^S	-0.09	-0.28	0.04	0.16
μ^B	0.05	0.02	-0.01	-0.04
σ^S	0.14	0.14	0.11	0.17
σ^B	0.04	0.03	0.04	0.05
ρ	-0.39	-0.25	-0.12	-0.43

- We can estimate the transition matrix $\boldsymbol{Q}$, the dynamics between regimes and the next level of stock-bond correlations.

Hierarchical clustering (II)



- The goal : Define the notion of a regime in a non-naive way.

- What we have : High-dimensional data i.e.

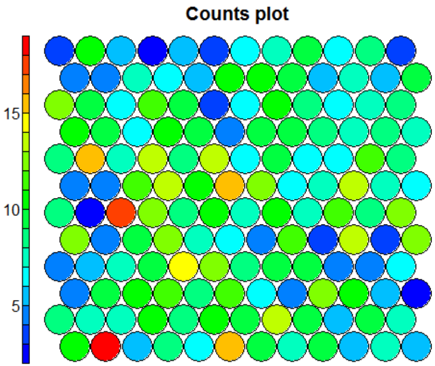
$$\Big(\mathrm{Date}, \mathrm{S.returns}, \mathrm{B.returns}, \mathrm{S.volume}, \mathrm{B.volume},$$
$$\mathrm{Vol.Index}, \Delta\mathrm{GDP}, \Delta\mathrm{Inflation}, \Delta\mathrm{InterestRate}\Big)$$

- How are we going to do this ?
- Check for clustering
- Problem : High-dimensional data
- Solution : SOM

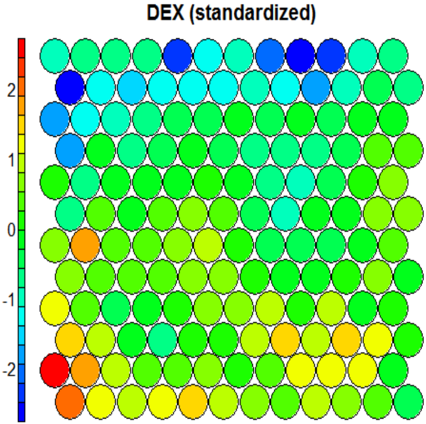
Input :

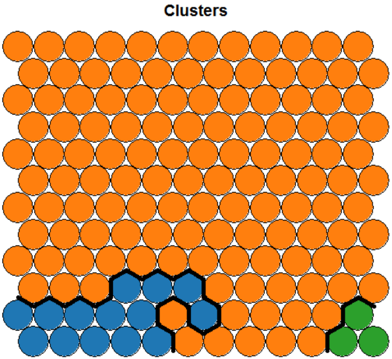
$$\Big(\text{Date}, \text{Sprice}, \text{Bprice}, \text{Svolume}, \text{Bvolume},$$
$$\text{Vol.Index}, \Delta\text{GDP}, \Delta\text{Inflation}, \Delta\text{InterestRate}\Big)$$

Output :



Heatmaps :




Clusters

Concluding remarks :

- SOM is extremely effective when the data is not time dependent i.e. (Hair, Eye color, Height, Weight).
- SOM is easy to implement.
- SOM cannot understand time dependence easily.

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