Modelling and Specifying Dispersive Laser Cavities

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Problem posed by Bryan Burgoyne, Genia Photonics

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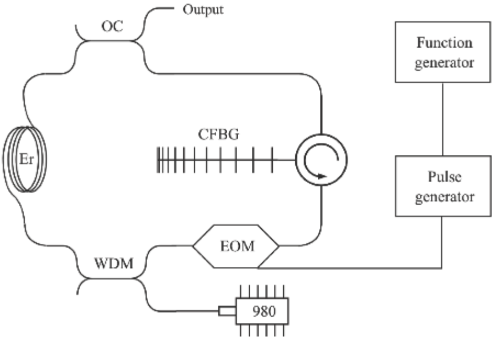
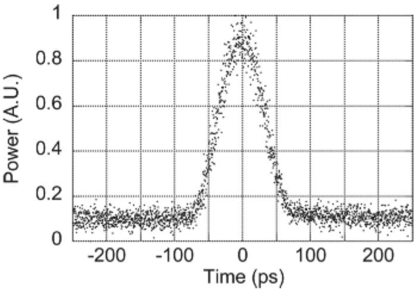
Outline

Tuneable Lasers

The laser is a dispersion-tuned, actively mode-locked, fibre laser. Its principal characteristics are the electronic generation of the laser pulses using a time modulation and the presence of a highly dispersive element in the cavity. The dispersive element causes the various possible wavelengths to have different propagation times in the cavity. By electronically selecting the repetition rate of the pulse using the time modulation, we effectively select the wavelength.

Tuneable Lasers





EOM: electrooptics modulator; OC: optical coupler; WDM: wavelength division multiplexer; CFBG: chirped fibre Bragg grating.

Wave-Breaking

All components of the cavity are in optical fibre, so the laser pulses always propagate in glass. The optical nonlinearity of the glass must be taken into account since the pulse power is high and the fibre several meters long.

The interplay between the dispersive element, the time modulation window, and the fibre nonlinearity deforms the pulse envelope and generates additional frequencies. Under some conditions to be determined, this interplay leads to the breaking of the pulse envelope.

Wave-Breaking

- The exact mechanism is not entirely clear.
- Nonlinearities generate new frequency components around the pulse that are shifted in time by the dispersion.
- The frequencies at which these components are generated depend on the pulse profile.
- Interference results when different frequencies coexist in the pulse at the same time.
- Fringes appear on the pulse envelope, thus changing the nonlinear phase shift and leading to the generation of more frequency components and the breaking of the pulse.
- The time modulation can remove some of these frequencies but it also shapes the pulse, making the exact behaviour of the laser difficult to predict.

Measuring the effect of the various experimental parameters is prohibitively expensive and difficult to implement. We need high-resolution models that predict what is observed experimentally.

Questions to the group

- Why does the wave break?
- What are the ranges of the effective parameters that give a wave with certain properties?
- How do the parameters interact with one another?
- Can a particular wave profile be specified?

Average Model

If we ignore nonlinearities in the fibre, a typical technique to model the laser is to write a single equation governing the behaviour of the average electric field. We assume a small perturbation while the wave propagates in the cavity. This yields an expression of the form

$$\frac{\partial A}{\partial z} = -\mathrm{i}\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \frac{\epsilon}{2}T^2 A + \frac{g}{2}A,$$

where A is the complex amplitude of the electric field, $|A|^2$ is the optical power, T denotes the time in the frame of the moving pulse, and z corresponds to the number of roundtrips in the cavity.

β_2 is the net second order of the dispersion, ϵ is related to the bandwidth of the modulation, and g is the net gain of the cavity.

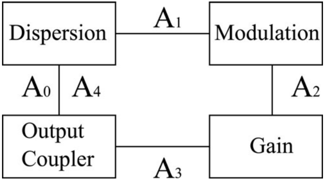
Average Model

Difficulties with the Average Model

- There is no separation between the dispersion of the fibre and the dispersion of the CFBG.
- The gain does not saturate with the pulse energy.
- The fibre is inherently nonlinear and for the lengths under consideration, it modifies the phase while hardly changing the energy.

Breaking the device into the various discrete components joined by fibre has already been shown to be much better at reproducing the experimental results (especially at higher pulse energies).

Hybrid Model



Properties of the Hybrid Model

- Gain saturates with the pulse energy.
- Modulation reflects the profile being used experimentally.
- The fibre is modelled with a nonlinear phase evolution and minimal dispersion.
- Dispersion corresponds to a known built-in frequency dependence.

Component: Saturated Gain

The amplitude in the gain component satisfies the following PDE.

$$\frac{\partial A}{\partial z} = \frac{g_0 A}{2\left(1 + E/E_{\mathrm{sat}}\right)}, \qquad E = \int_{-\infty}^{\infty} |A|^2 \,\mathrm{d}\,T$$

E is the net energy of the pulse; g_0 and E_{sat} are parameters. This component affects the amplitude of the pulse but preserves the phase.

Finding an equation for the energy yields the implicit solution that can be inverted using the Lambert W-function.

$$\frac{E}{E_{\mathrm{sat}}}\,\mathrm{e}^{E/E_{\mathrm{sat}}} = \frac{E_0}{E_{\mathrm{sat}}}\,\mathrm{e}^{E_0/E_{\mathrm{sat}}}\,\mathrm{e}^{g_0 z}, \quad \frac{E(z)}{E_{\mathrm{sat}}} = W_0\left(\frac{E_0}{E_{\mathrm{sat}}}\,\mathrm{e}^{E_0/E_{\mathrm{sat}}}\,\mathrm{e}^{g_0 z}\right).$$

Component: Saturated Gain

Returning to the amplitude we find

$$A(z,T) = A_0(T)\sqrt{\frac{E_{\text{sat}}}{E_0}}\sqrt{W_0\left(\frac{E_0}{E_{\text{sat}}}\,\mathrm{e}^{E_0/E_{\text{sat}}}\,\mathrm{e}^{g_0 z}\right)},$$

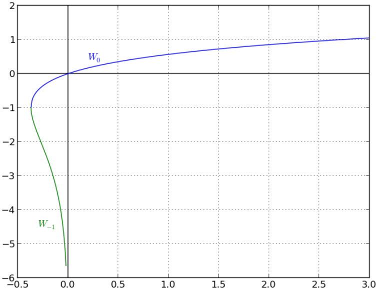
with $A_0(T)$ being the input pulse profile at $z = 0$.

The net effect of the gain component as a map from the input $A(T)$ to the output $G[A](T)$ is

$$G[A] = A\sqrt{\frac{E_{\text{sat}}}{E}}\sqrt{W_0\left(\frac{E}{E_{\text{sat}}}\,\mathrm{e}^{E/E_{\text{sat}}}\,\mathrm{e}^{g_0 \ell_G}\right)}, \quad E = \int_{-\infty}^{\infty} |A|^2 \,\mathrm{d}T,$$

where ℓ_G is the length of the gain component.

Component: Saturated Gain/Loss



Loss

The whole circuit has a net loss described by the map $L[A] = A\mathrm{e}^{-\alpha L/2}$, where α is a loss coefficient and L is the length of the whole circuit.

Component: Modulation

The modulation component simply multiplies A by a specified function of $\mathcal{T}$:

$$M[A] = \mathcal{T}(T)A,$$

where the transfer function is assumed to be given by

$$\mathcal{T}(T) = \frac{1}{2} - \frac{\nu}{2}\cos\left(\mu\pi \mathrm{e}^{-T^2/T_M^2}\right),$$

with T_M being the width of the modulation window. The parameters μ and ν are both equal to one in the ideal case.

Component: Fibre (no dispersion)

Dispersion is negligible when compared with the dispersion of the grating. Neglecting the fibre dispersion gives a fibre amplitude that satisfies

$$\frac{\partial A}{\partial z} = \mathrm{i}\gamma |A|^2 A,$$

where γ is in principle a known parameter. In this limit the fibre only affects the phase and the solution for A is

$$A(z, T) = A_0(T)\mathrm{e}^{\mathrm{i}\gamma z |A_0(T)|^2},$$

where $A_0(T)$ is the profile input at $z = 0$. Therefore the net effect of the fibre component is described by the map

$$F[A] = A\mathrm{e}^{\mathrm{i}\gamma \ell_F |A|^2},$$

where ℓ_F is the length of the fibre.

Component: Dispersion

Dispersion is specified by its action in the frequency domain. If we define the Fourier transform of A as

$$\mathcal{F}[A](\Omega) = \int_{-\infty}^{\infty} A(T)\mathrm{e}^{\mathrm{i}\Omega T}\,\mathrm{d}\,T$$

then the effect on $\mathcal{F}[A]$ is given by

$$D : \mathcal{F}[A] \mapsto \mathcal{F}[A]\mathrm{e}^{\mathrm{i}\bar{\beta}_2\Omega^2},$$

where $\bar{\beta}_2$ is a known dispersion coefficient. This can be expressed as a convolution of A with the action of the dispersion, yielding

$$D[A] = \frac{\mathrm{e}^{\mathrm{i}\pi/4}}{2\sqrt{\pi\bar{\beta}_2}} \int_{-\infty}^{\infty} A(\tau)\mathrm{e}^{-\mathrm{i}(\tau-T)^2/4\bar{\beta}_2}\,\mathrm{d}\tau.$$

Non-dimensionalization

From the structure of the blocks the relevant variables are time, energy, and amplitude of the pulse with the various length scales implicit in the parameters. We non-dimensionalize with

$$T = T_M \hat{T}, \qquad E = E_{\mathrm{sat}} \hat{E}, \qquad A = \sqrt{\frac{E_{\mathrm{sat}}}{T_M}}\, \hat{A}.$$

Dropping the hats, the normalised maps take the following form.

$$G[A] = \frac{A}{\sqrt{E}} \sqrt{W_0\left(aE\mathrm{e}^{E}\right)}, \qquad L[A] = Ah,$$

$$M[A] = A\left[\frac{1}{2} - \frac{\nu}{2}\cos\left(\mu\pi\mathrm{e}^{-T^2}\right)\right], \qquad F[A] = A\mathrm{e}^{\mathrm{i}b|A|^2},$$

$$D[A] = \frac{\sigma\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{\pi}} \int_{-\infty}^{\infty} A(\tau)\mathrm{e}^{-\mathrm{i}\sigma^2(\tau-T)^2}\, \mathrm{d}\tau.$$

Dimensionless Parameters

The problem contains four remaining dimensionless parameters,

$$(\text{Gain}):\quad a = \mathrm{e}^{g_0 \ell_G} \sim 30, \qquad (\text{Loss}):\quad h = \mathrm{e}^{-\alpha L/2} \sim 0.1,$$

$$(\text{Fibre}):\quad b = \frac{\gamma \ell_F E_{\text{sat}}}{T_M} \sim 10, \quad (\text{Dispersion}):\quad \sigma = \frac{T_M}{2\sqrt{\bar{\beta}_2}} \sim 5.$$

Each of these can be individually tuned by varying the length and material properties of the various elements within the laser cavity.

Problem Restatement

Each block having been specified, the circuit consists of concatenating the blocks in a given order, for example

$$\cdots \text{ Gain } \rightarrow \text{ Disp. } \rightarrow \text{ Fibre } \rightarrow \text{ Loss } \rightarrow \text{ Mod. } \rightarrow \text{ Gain } \cdots$$

and looking for profiles that are invariant with respect to this process. The outcome of the entire circuit is described in terms of the blocks as

$$\mathcal{G}[A] = M[L[F[D[G[A]]]]]$$

and a steady solution consisting of a fixed point of the map $A \mapsto \mathcal{G}[A]$. Moreover we expect the "wave breaking" instability observed in experiments to correspond to a loss of stability of the fixed point.

Stationary Phase

The dimensionless parameters b and σ (respectively associated with the fibre and the dispersion) are consistent with the experimental observation that the phase of A changes rapidly as a function of T while the amplitude varies slowly. This can be exploited by using the method of stationary phase to approximate the convolution.

Letting $A(T) = |A(T)|\mathrm{e}^{\mathrm{i}\sigma^2\psi(T)}$, we find in the limit (as $\sigma \to \infty$) that the leading-order of $D[A]$ takes the form

$$D[A] \sim \left(1 - \frac{\psi''(\tau_*)}{2}\right)^{-1/2} A(\tau_*)\mathrm{e}^{-\mathrm{i}\sigma^2(\tau_* - T)^2},$$

where

$$\psi'(\tau_*) = 2(\tau_* - T).$$

Results 1: Laurent

Simple iteration using an FFT

- Use an FFT to compute the effect of the dispersion:

$$D(A) = \mathcal{F}^{-1}\left(e^{\frac{i\omega^2}{4\sigma^2}}\hat{A}(\omega)\right).$$

- Explore the effect of changes in the parameters on the shape of the resulting pulse.
- Determine ranges of the parameters where the pulse is stable.

Special care has been taken concerning the correct multiplication in the frequency domain.

Results 1

The gain, modulation, fibre, and loss transfer functions are straightforward:

$$G(A) = \sqrt{\frac{W_0(aEe^E)}{E}}A, \qquad M(A) = \frac{1}{2}(1 - \cos(\pi e^{-T^2}))A$$

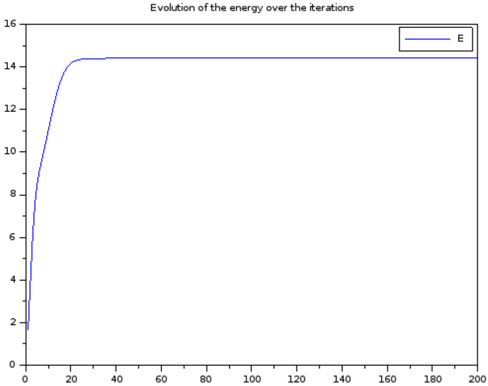
$$F(A) = Ae^{ib|A|^2}, \qquad L(A) = hA.$$

A is discretized over a numerical time window $(-T_m, T_m)$ with N points. The energy E is computed as a simple trapezoidal numerical integration of $|A|^2$ on the time window.

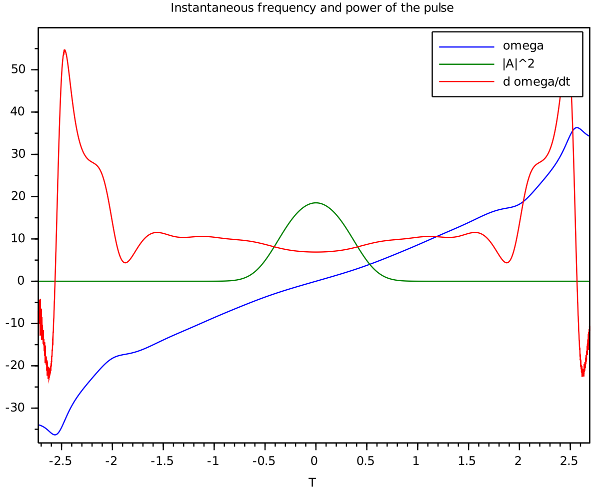
Number of points in the discretization: 16384. Size of the simulation window: $(-8, 8)$. Nominal parameters: $a = 30$, $h = 1$, $\sigma = 5$.

No breaking

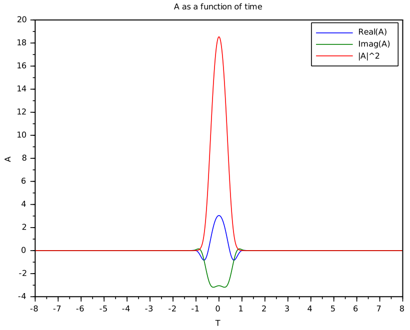
Parameter choice: $b = 0.01$ (effective fibre length). The observed kurtosis is 2.645 (less peaked than a Gaussian).

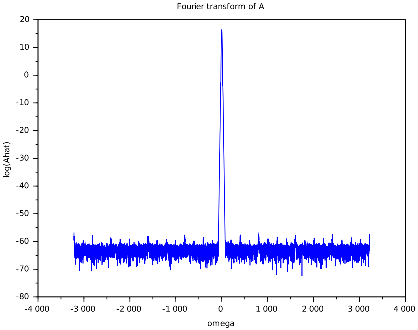


No breaking


Instantaneous frequency and power of the pulse
omega
|A|^2
d omega/dt
50
40
30
20
10
0
-10
-20
-30
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
T

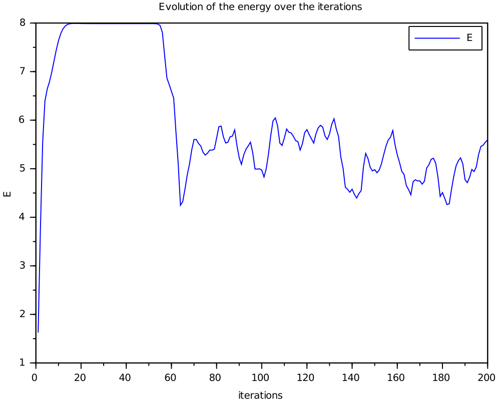
No breaking


A as a function of time
Real(A)
Imag(A)
|A|^2
A
T
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

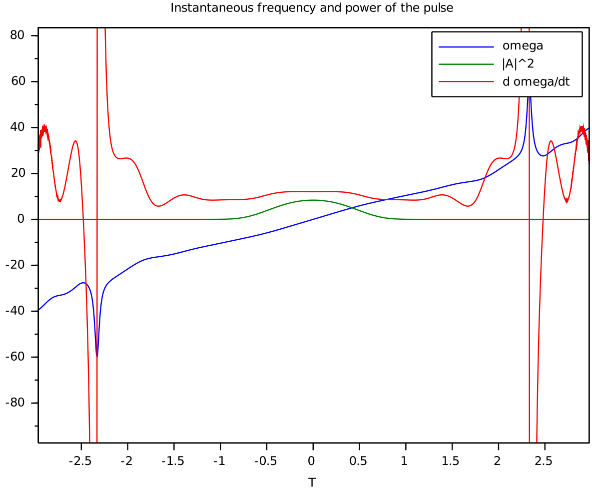

Fourier transform of A
log(Ahat)
omega
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-4 000 -3 000 -2 000 -1 000 0 1 000 2 000 3 000 4 000

Breaking

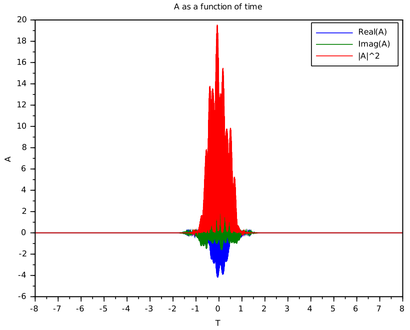
Parameter choice: $b = 0.1$ (effective fibre length). The observed kurtosis is 2.37 before the onset of instability.



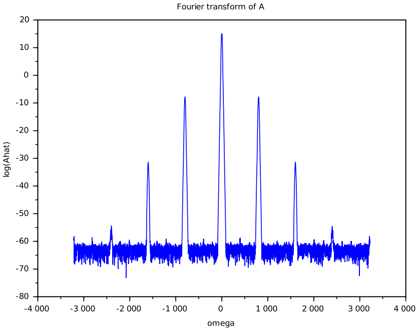
Breaking


Instantaneous frequency and power of the pulse
omega
|A|^2
d omega/dt
80
60
40
20
0
-20
-40
-60
-80
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
T

Breaking



A as a function of time
Real(A)
Imag(A)
|A|^2
A
T



Fourier transform of A
log(Ahat)
omega

Onset of Breaking

Some observed limits of stability

For $a = 30$, $h = 1$, we have observed the following upper bounds of stability for b at a given σ:

- $\sigma = 2$: $b \lesssim 0.008$
- $\sigma = 5$: $b \lesssim 0.015$
- $\sigma = 10$: $b \lesssim 0.02$

Caution!

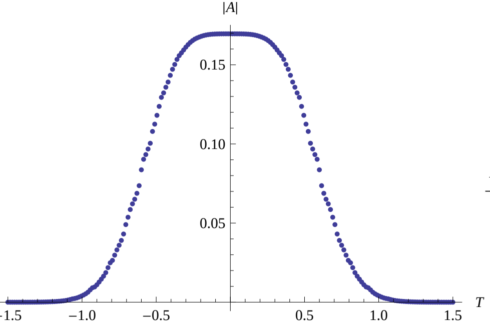
Some features of the breaking waves, such as the growth of frequencies linked to the size of the domain of the FFT, might influence the numerical results obtained.

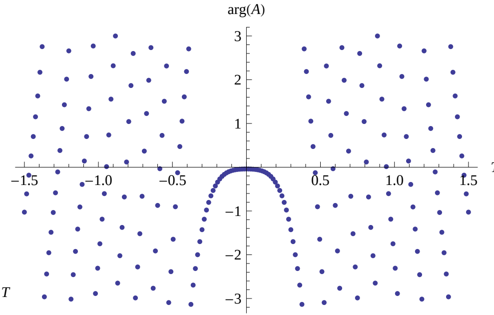
Results 2

Approximation using stationary phase

- Parameter values: $a = 30, b = 10, h = 0.25, \sigma = 5$.
- Uses the stationary phase representation of the dispersion.
- Assumes that ψ'' is essentially constant so that it does not matter where it is evaluated.
- Gain, loss, modulation, and fibre blocks are as above.

Results 2





Conclusions

- The various schemes all show that there is a set of parameters that do indeed mimic the shape of the pulses observed in the laboratory.
- Pulses can be shaped by modifying the parameters but some work is still needed to determine the viable range for preventing the wave from breaking.
- The proposed hybrid model must be cleaned up and details must be taken care of, but the results are tantalizing.

New Directions

- Considering that fibre and the dispersion we see that

$$F[A] = A\mathrm{e}^{\mathrm{i}b|A|^2}, \qquad D[A] \sim A(\tau_*)\mathrm{e}^{-\mathrm{i}\sigma^2(\tau_* - T)^2}.$$

  Ignoring higher order effects, the shape of the profile is driven by $\mathrm{e}^{-\mathrm{i}\sigma^2(\tau_* - T)^2}$, which comes from the dispersion element. Nonlinear effects in the fibre could be compensated by a modified phase resulting from the dispersion element.
- Determine the "best" pulse shape so as to obtain the most linear chirp possible. This will lead to an efficient pulse compression.
- Spreading the pulse out and modulating to clip the pulse generates resonances in the spectrum, giving information about the natural resonances in the cavity. This information may be useful.

Thank-you!