Nonlinear Continuous Deformation of an Image Based on a Set of Coplanar Straight Lines

P. Armand
D. Brunet
P. Chaput
A. Kiselev
O. Marcotte
A. Morin-Duchesne
D. Orban
N. van Omme
and
V. Zalzal from MATROX

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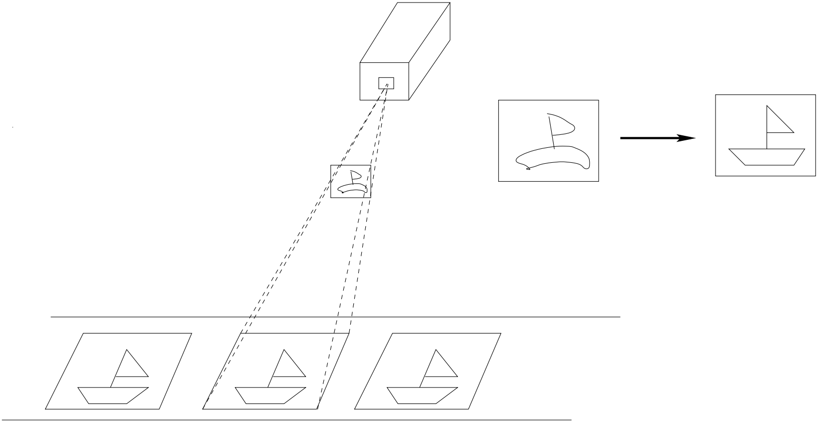
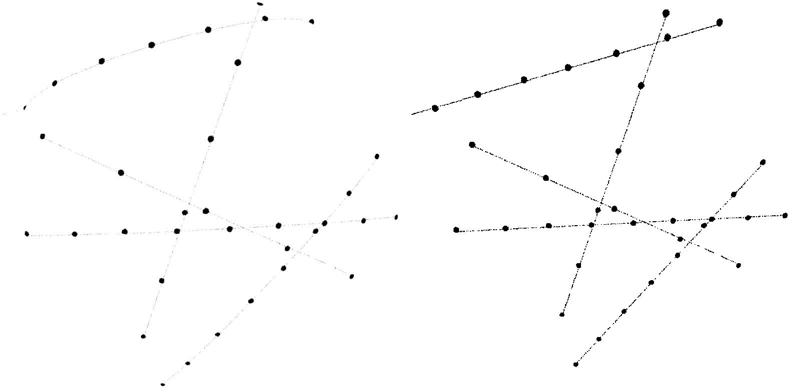
Problem Description



Requirements

- A high precision
- An image reconstruction taking less than 1 second

An Example



N : number of lines (at most 30)

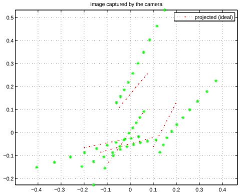
n_i : number of points in the i^{th} line ($8 \leq n_i \leq 25$)

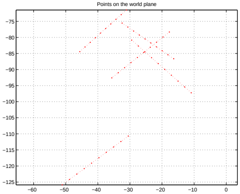
Assumptions

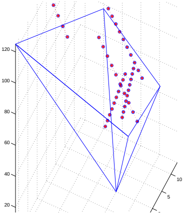
- Thin (or 2-dimensional) objects
- Automatic calibration
- Pinhole model
- Radial distortion ($r_d \approx r_u \left(1 + K r_u^2\right)$, where $K \sim 10^{-3}$)

Problem Decomposition

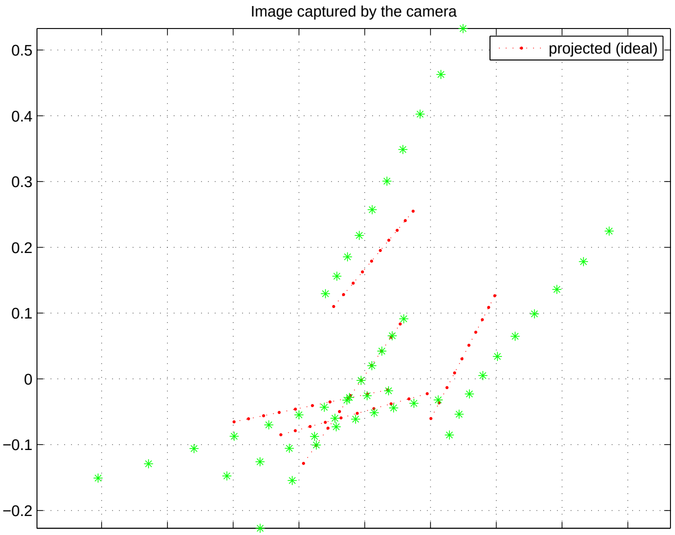
- Stage I: Eliminate distortion (Algorithms Dist1 and Dist2)
- Stage II: Rectify perspective (Algorithms Persp1 and Persp2)



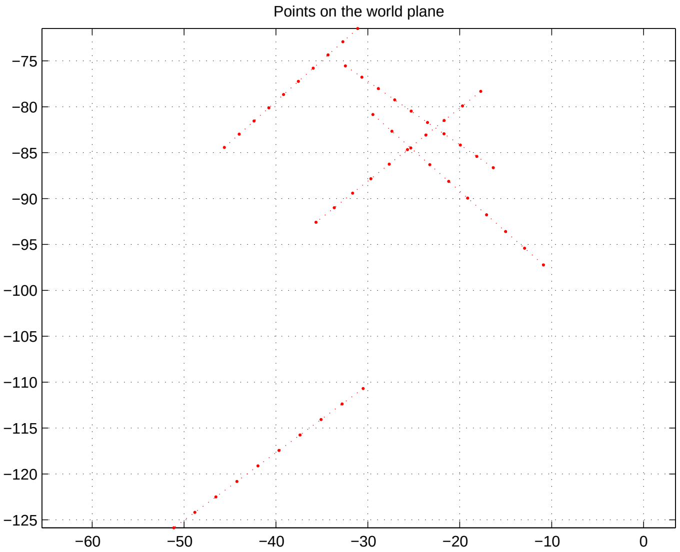




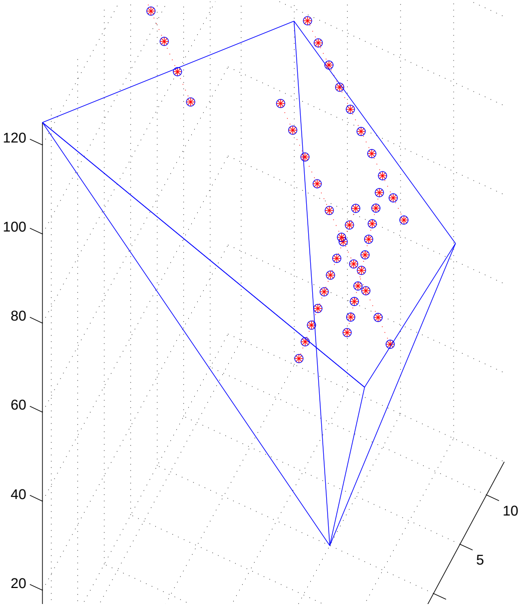
Points in the Focal Plane



Points in the World Plane



Reconstructed Image



Algorithm Dist1

g represents a radial distortion and (r, θ) a point in the plane

$$g(r, \theta) = (r(1 + Kr^2), \theta)$$

$d(.,.)$ denotes the Euclidean distance between two points.

$$d((u, v), (u', v')) = \sqrt{(u - u')^2 + (v - v')^2}$$

- (x_j^i, y_j^i) denotes the j^{th} point on the i^{th} straight line (obtained from a line in the world plane by projection on the camera plane).
- $(\bar{x}_j^i, \bar{y}_j^i)$ denotes the j^{th} point on the i^{th} distorted line in the camera plane.

Algorithm Dist1 (the Model)

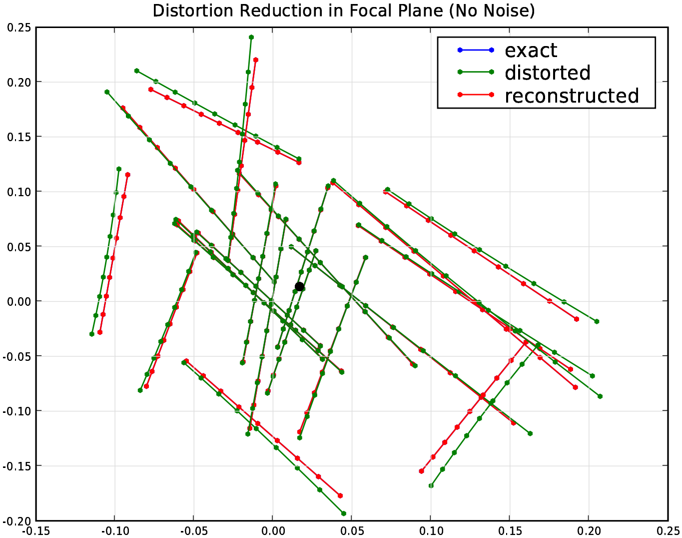
$$\text{minimize } \sum_{i=1}^{N} \sum_{j=1}^{n_i} \left(d \left(g \left(x_j^i, y_j^i \right), \left(\bar{x}_j^i, \bar{y}_j^i \right) \right) \right)^2$$

such that

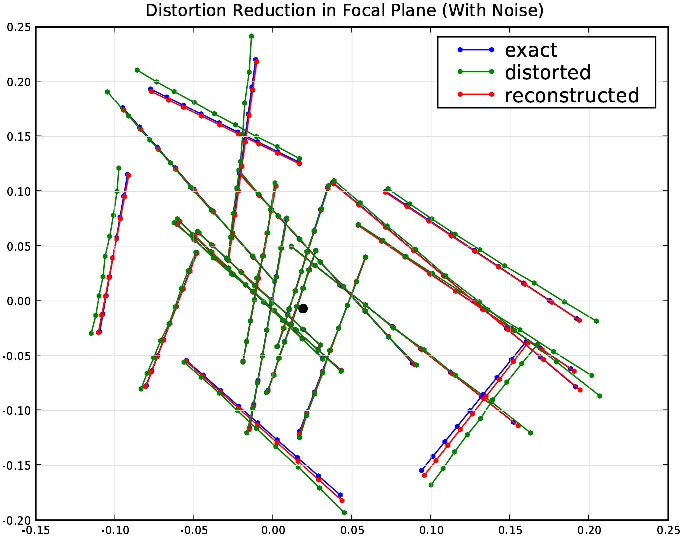
$$\frac{\begin{pmatrix} x_{j+1}^i - x_j^i \\ y_{j+1}^i - y_j^i \end{pmatrix}}{d\left(\left(x_{j+1}^i, y_{j+1}^i \right), \left(x_j^i, y_j^i \right) \right)} = \frac{\begin{pmatrix} x_j^i - x_{j-1}^i \\ y_j^i - y_{j-1}^i \end{pmatrix}}{d\left(\left(x_j^i, y_j^i \right), \left(x_{j-1}^i, y_{j-1}^i \right) \right)}$$

for all (i, j)

Results for Algorithm Dist1 (I)



Results for Algorithm Dist1 (II)

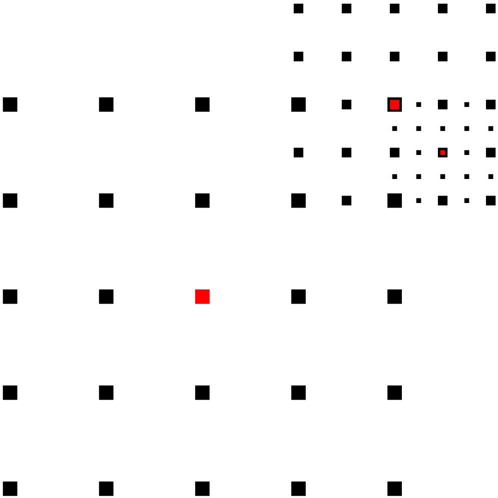


Results for Algorithm Dist1 (III)

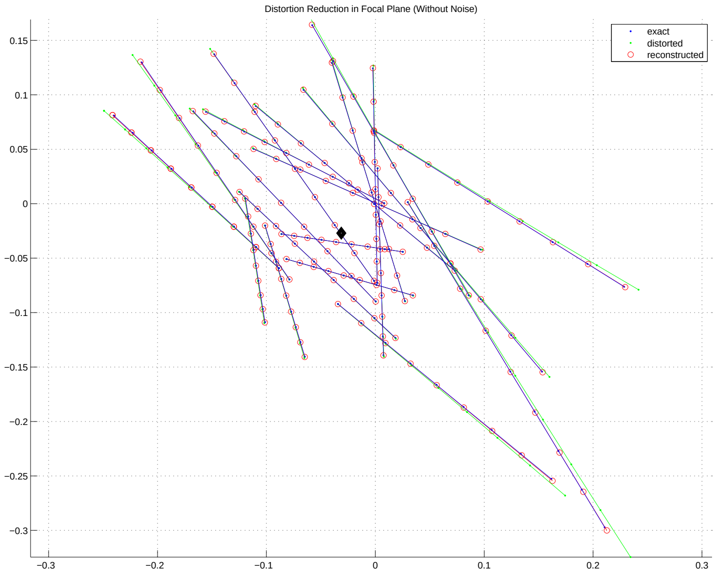
	Without Noise	With Noise
Time	0.52 s	0.36 s
Iterations	54	38
Rel. error	$(8.4 \cdot 10^{-5}; 1.2 \cdot 10^{-4})$	$(3.7 \cdot 10^{-2}; 3.3 \cdot 10^{-2})$
(x_0, y_0)	$(1.7 \cdot 10^{-2}; 1.4 \cdot 10^{-2})$	$(1.9 \cdot 10^{-2}; -7.0 \cdot 10^{-3})$
K	2.3006444	2.0012845

Real data: $(x_0, y_0) = (0.0168; 0.0136)$ and $K = 2.31$

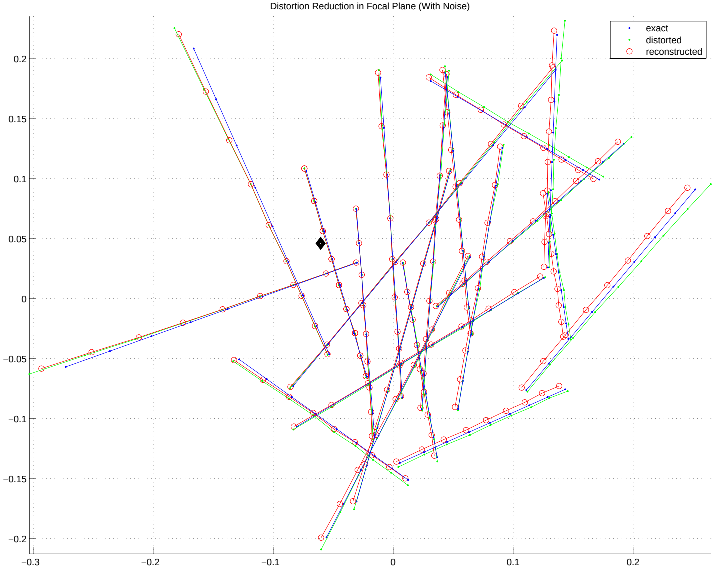
Algorithm Dist2



Results for Algorithm Dist2 (I)



Results for Algorithm Dist2 (II)



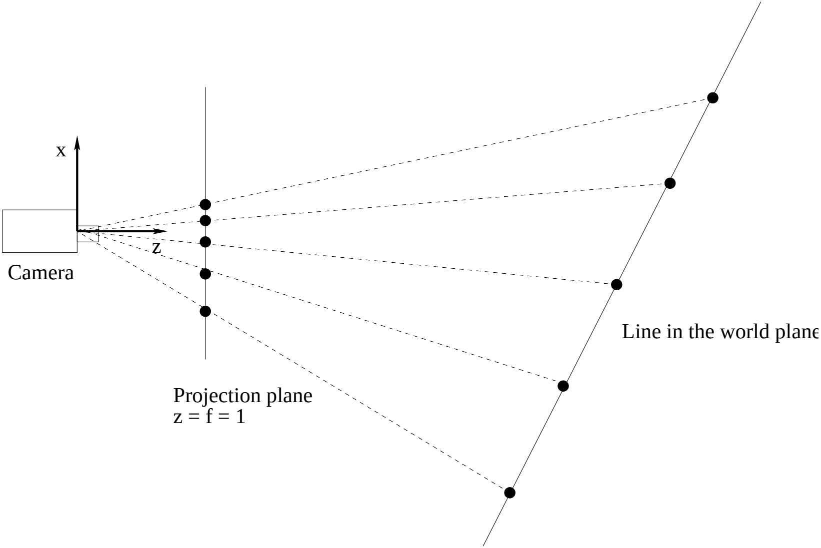
Results for Algorithm Dist2 (III)

	Without Noise	With Noise
Time	2.9s	2.9s
Rel. error	$(8 \cdot 10^{-3}; 4 \cdot 10^{-3})$	$(2.898 \cdot 10^{-3}; 5.117 \cdot 10^{-2})$
(x_0, y_0)	$(-0.0312387; -0.027)$	$(0.031676; 0.00994148)$
K	0.6679	0.66796

Real data

	Without Noise	With Noise
(x_0, y_0)	$(-0.026966; -0.02456)$	$(-0.060403; 0.046148)$
K	0.7624	1.02918

Perspective: Illustration



Algorithm Persp1

$$\text{minimize} \sum_{i=1}^{N} \sum_{j=1}^{n_i} \left(r_{ij}^2 + e_{ij}^2\right)$$

such that

$$\begin{pmatrix} x_j^i \\ y_j^i \\ z_j^i \end{pmatrix} - \begin{pmatrix} x_{j+1}^i \\ y_{j+1}^i \\ z_{j+1}^i \end{pmatrix} = \begin{pmatrix} x_{j+1}^i \\ y_{j+1}^i \\ z_{j+1}^i \end{pmatrix} - \begin{pmatrix} x_{j+2}^i \\ y_{j+2}^i \\ z_{j+2}^i \end{pmatrix} + r_{ij}$$

$$a x_j^i + b y_j^i + c z_j^i = 1 + e_{ij}$$

$$x_j^i = \bar{x}_j^i z_j^i, \;\; y_j^i = \bar{y}_j^i z_j^i, \;\; z_1^1 = 1$$

Algorithm Persp2

$$\bar{x}_j^i = \frac{x_j^i}{z_j^i}, \;\; \bar{y}_j^i = \frac{y_j^i}{z_j^i}$$

$$\begin{pmatrix} x_{j+1}^i \\ y_{j+1}^i \\ z_{j+1}^i \end{pmatrix} - \begin{pmatrix} x_j^i \\ y_j^i \\ z_j^i \end{pmatrix} = \begin{pmatrix} \Delta_x^i \\ \Delta_y^i \\ \Delta_z^i \end{pmatrix}$$

$$\bar{\Delta}_x^i = \frac{\Delta_x^i}{z_1^i}, \;\; \bar{\Delta}_y^i = \frac{\Delta_y^i}{z_1^i}, \;\; \bar{\Delta}_z^i = \frac{\Delta_z^i}{z_1^i}, \;\; a\bar{\Delta}_x^i + b\bar{\Delta}_y^i + c\bar{\Delta}_z^i = 0, \;\; a = 1$$

$$-(j-1)\bar{\Delta}_x^i + (j-1)\bar{x}_j^i\bar{\Delta}_z^i = \bar{x}_j^i - \bar{x}_1^i$$

$$-(j-1)\bar{\Delta}_y^i + (j-1)\bar{y}_j^i\bar{\Delta}_z^i = \bar{y}_j^i - \bar{y}_1^i$$

Illustration: noise = 0.0

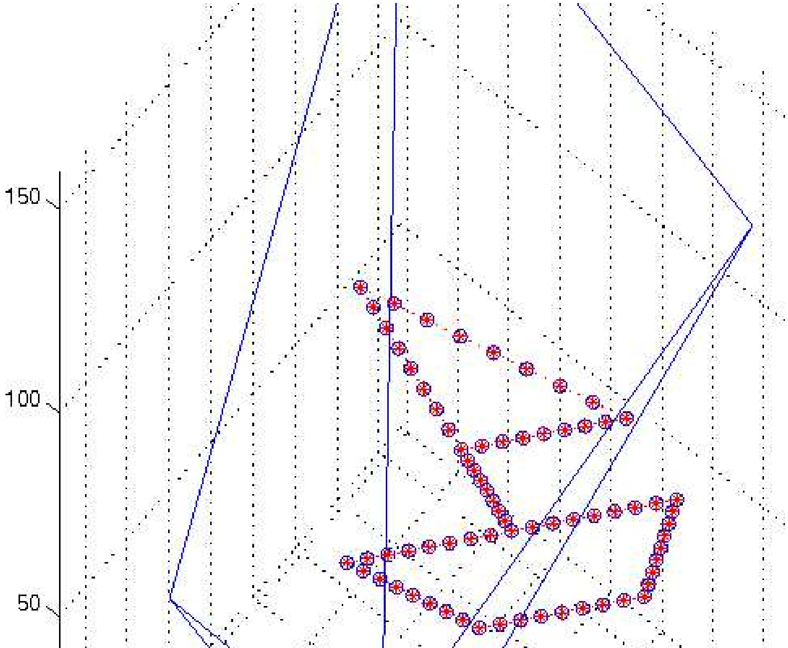


Illustration: noise = 0.5

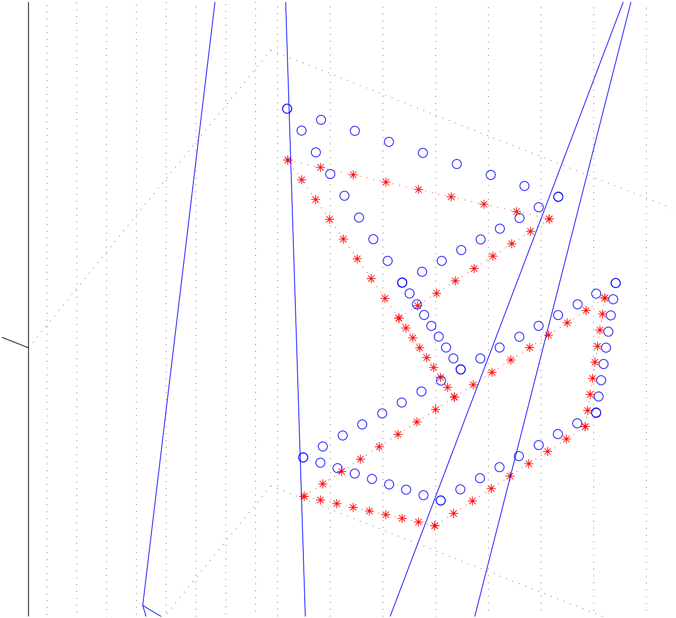


Illustration: noise = 1.0

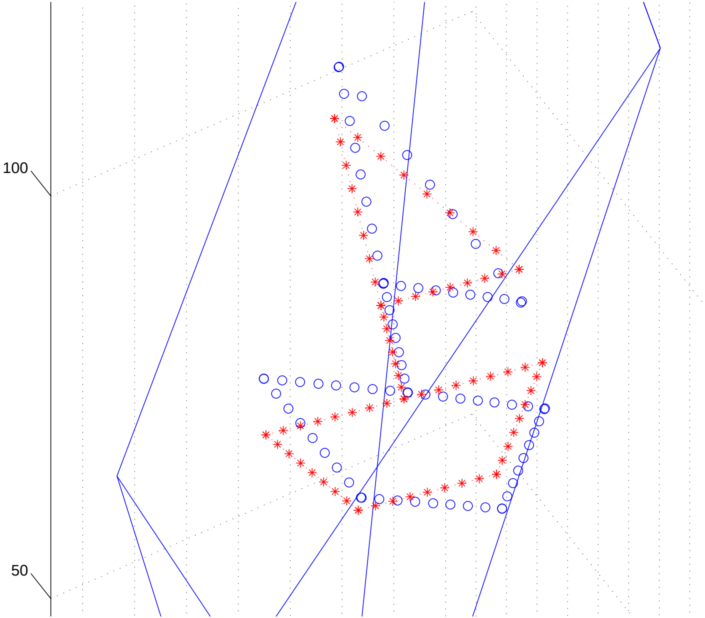


Illustration of Dist2 and Persp2 (I)

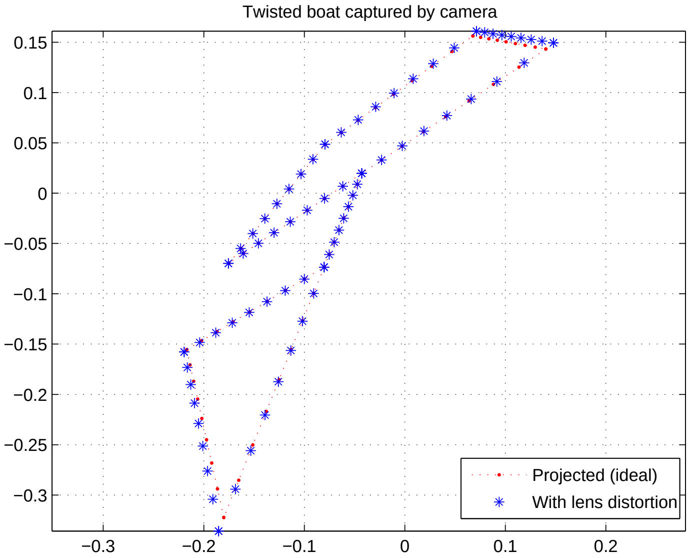
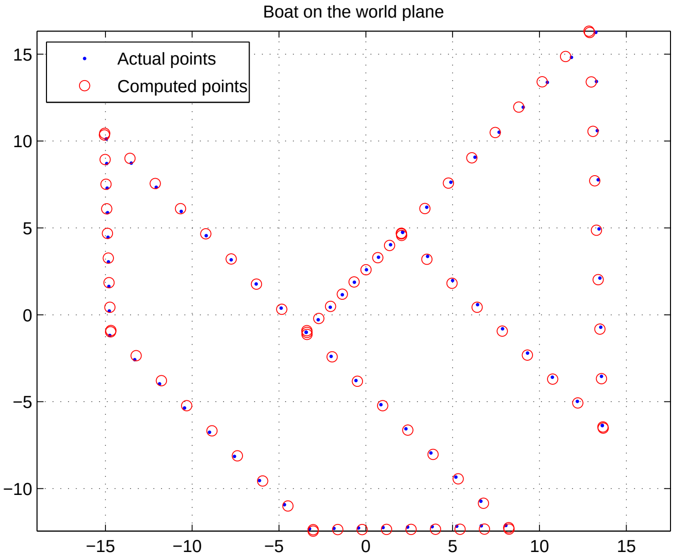
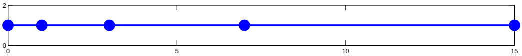


Illustration of Dist2 and Persp2 (II)



Future Research

- Modify the distortion elimination algorithm to take into account the fact that the straight line is a projection



- De-noise before removing distortion

- Investigate the influence of noise distribution

- Weaken the assumptions