MAINTENANCE PLANNING NEEDS AT SOCIÉTÉ DE TRANSPORT DE MONTRÉAL

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THE TEAM !

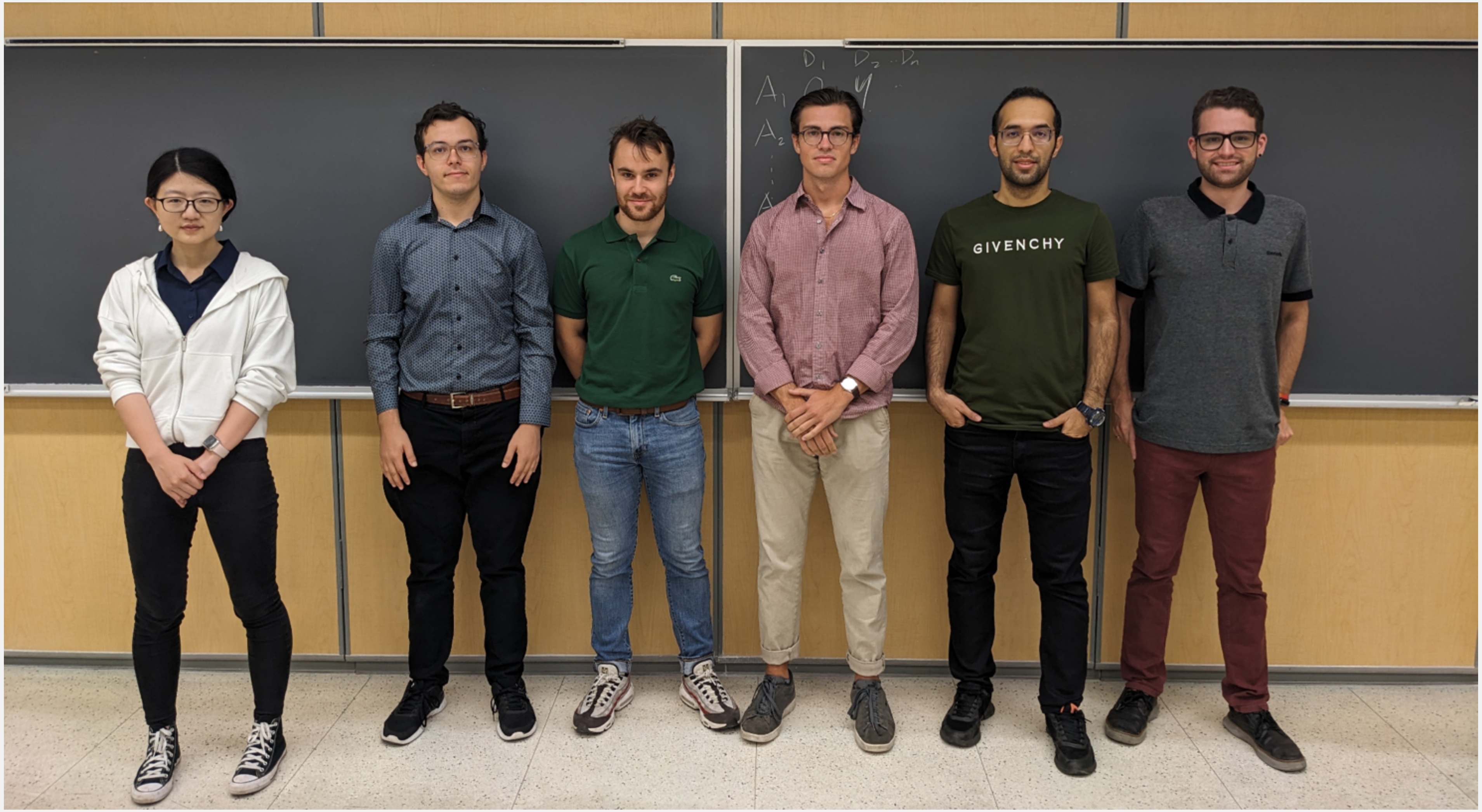


TABLE OF CONTENT

PROBLEM DEFINITION

PROBLEM SUBMITTED

Spare part classification based on « demand profile »

- Manage bundles of items rather than individual item to tackle workload issues,
- Identify the most suitable storage and procurement strategy for each of these "bundles of items"
- Apply similar storage and procurement strategies within these bundles.

Identify the most efficient forecasting method for each of the group of items with similar demand profiles.

- Conventional statistical forecasting models work very well for parts with relatively stable consumption patterns. For parts with more sporadic and lumpy demand profiles, these models are less effective and shouldn’t be used.
- Researchers such as Croston, Boylan and Syntetos have extensively studied these statistical models. The STM would like to evaluate their performance on its parts catalog.

PROBLEM DEFINITION

PROBLEM ADDRESSED

Since the STM's ultimate goal is to improve the management of its spare parts inventories, we will be looking to find a model that will suggest **quantities of items to purchase on a specific time** considering an observed demand for spare parts in the past data.

“The right part, at the right place, at the right time, minimizing the inventory costs given a time period”

DATASET

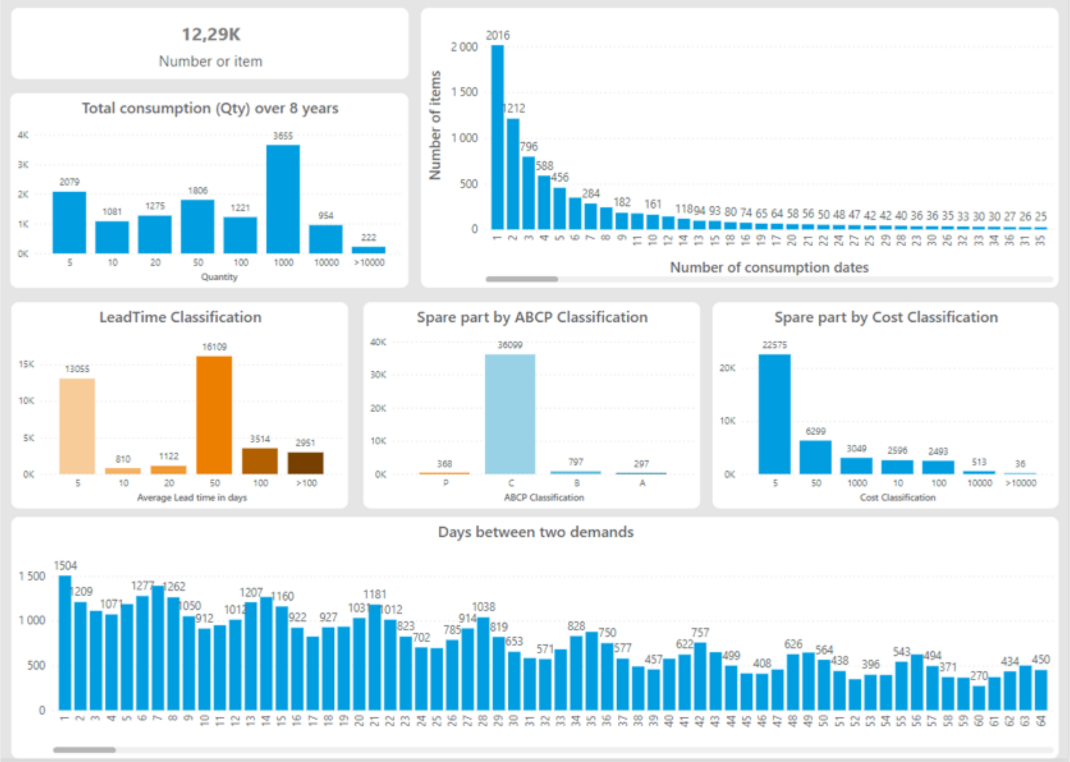
METRO DATASET

Item consumption matrix from 01-Jan-2015 to 01-Aug-2023

- Up to **8 years** of historic data of spare consumption
- **No change in the asset fleet** over the analysis horizon
  - No new or scrapped trains, just an aging fleet.
- Maintenance sector with the organization's **most qualitative data**
- **First Assumption :** No seasonal effect in demand,
  - Less likely to feel the seasonal effect of frost or heat in the tunnels than busses demand dataset

	01-Jan-2015 Day2 Day3 … Day N-1 01-Aug-2023
Item 1	0 0 1 0 … 0 3 1
Item 2	0 3 12 0 … 0 0 3
. . .	
Item 13k…	…. ….

DATASET



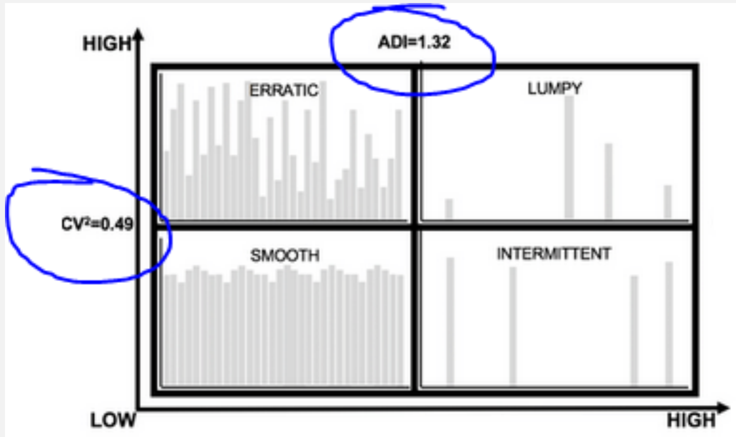
DESCRIPTIVE ANALYSIS

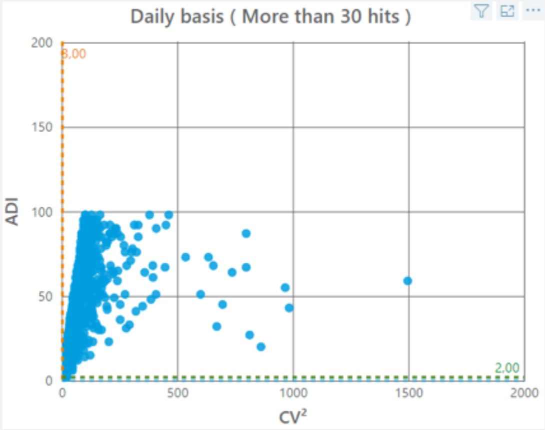
We used Power Bi **to navigate and drill** into the database in order to get quick insights of our dataset and get some orders of magnitude.

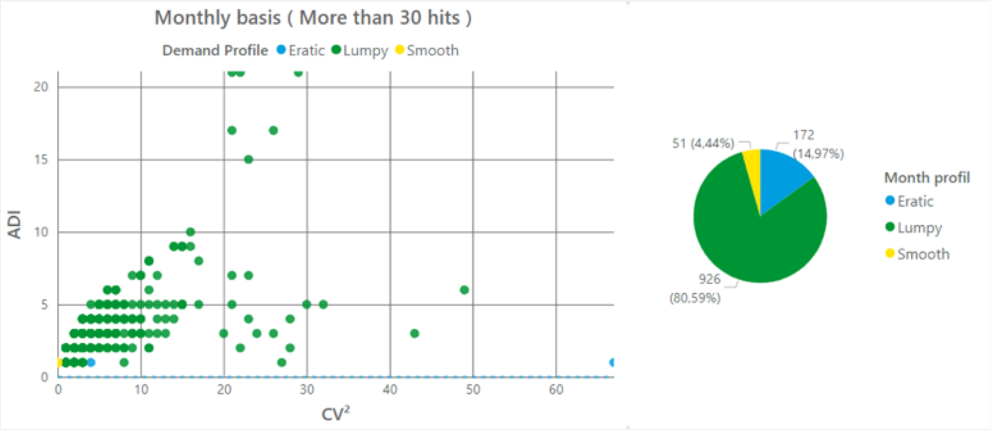
We made the assumption that we needed more than **X points** in the history to make predictions : cutoff point is to be determined.

DEMAND PROFILS

Evaluation of our demand characteristics in terms of frequency variability (Related to ADI) and quantity variability (Related to CV^2)



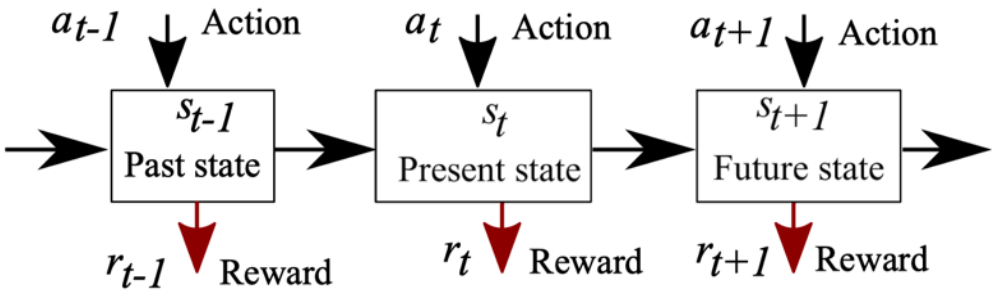




MARKOV DECISION PROCESS (MDP)

PROS

- A **Powerful and versatile** framework for wide applications to solve **sequential decision making problems under uncertainty**.
- **Balance** decisions between **short-term and long-term** rewards.
- Solve the problem in an **optimized manner** instead of using order-point heuristics.
- Useful to handle **complex scenarios** by exploiting past knowledge.



Sequential decision making process *[A Biswas et al., 2019]*

$$\max_{\pi} \mathbb{E}_{a\sim\pi(s)}\left[\sum_{t=1}^{T} r(s_t, a_t)\middle| s_0\right]$$

- $s \in S$ is a (finite) set of states
- $a \in A$ is a (finite) set of actions
- $r(s,a) \in R : S \times A \to \mathbb{R}$ is the reward (Applicable for **cost-minimization** setups as well)

TIME STEP & TIME HORIZON

FINITE HORIZON

Due to **non-stationarity** of the problem parameters and the demand variable, we consider finite-horizon setup where all the fixed parameters are **updated at the end** of the time horizon.

$$T_1 : t\,\{1, \dots, T\}$$

$$T_2 : t\,\{1, \dots, T\}$$

Recalculate the exact:

LT, ADI, SS

The **Lead time (LT) and Average Demand Interval (ADI)** are the main parameters by which we define the time steps (t) and the time horizon (T).
We set the **time step** to be greater than LT.
We set the **time horizon** to be : T = MAX (K ADI), K' >> 1

State Space & Action Space (For Each Article)

Articles state space (S)

*This state corresponds to the **stocked inventory** of the article at the beginning of the time step.*

$$S_t \in \{0, \ldots, s_{\max}\}$$

$s_{\max}$: maximum inventory

S_t : inventory at time t

Action space (A)

This space represents all the possible number of articles to order from a supplier at a time step. Some articles have a minimum (amin) and maximum (amax) amount that it is possible to order in

$$A_t^{LOW} \in \{0, a_{\min}, a_{\min} + 1, \ldots, a_{\max} - 1, a_{\max}\}$$

$$A_t^{\mathrm{HIGH}} \in \{0, \ldots, a_{\max}\}$$

$a_{\max}$: maximum quatity per order

$a_{\min}$: minimum quatity per order

A_t : action at time t

DEMAND DISTRIBUTION

DEMAND DISTRIBUTION

The demand distribution is important as it helps us in predicting the future behavior and be prepared.

Let N be the total number of the item used in the system.
For a given item, the probability of a demand is denoted by p (which is estimated using historical data).
We define D_t to be the random variable of the number of items at demand for the specific item at time t.
The distribution of the random variable D_t is given by the following distribution:

$$\mathbb{P}(D_t = k) = \binom{N}{k} p^k (1-p)^{N-k}$$

STATE DYNAMICS

STATE DYNAMICS

The state dynamics of our model is as follows:

$$S_{t+1} = S_t - D_t + A_t$$

Due to the randomness in the demand, the transition probability of moving from an inventory level to another one can be defined as follows:

$$\mathbb{P}(S_{t+1} = s' | S_t = s, A_t = a) = \mathbb{P}(D_t = \max(0, s + a - s'))$$

Cost function

$$\boxed{c(X_t, A_t, K) = Hc + IMc + SSc + POc}$$

Hc : **Holding cost ($)**
IMc : **Immobilisation cost ($)**
SSc : **Safety stock cost ($)**
POc : **Purchase order cost ($)**

Xt : Inventory
At : Action
K : All properties of every article
SS : Safety stock value

Holding cost : We don't want to hold unneeded items.

- We consider that the annual storage cost of an item corresponds to 20% of its price (order of magnitude).
- Volume has a significative influence on holding costs :

$$V_{max} = Max\ catalog\ volume\ item\ (m^3)$$

$$V = Item\ volume\ (m^3)$$

- Penalize the storage of parts with low criticalities :

$$CT = Criticity\ of\ the\ item$$

$$\begin{aligned} CT &= 1\ if\ the\ item\ is\ classified\ 'A'or'P' \\ &= 0\ if\ the\ item\ is\ classified\ 'B'or'C' \end{aligned}$$

$$Hc = \frac{0{,}2Pc}{365} * x_t * \frac{V}{V_{max}} * (2 - CT)$$

Cost function

Immobilization cost :

Today, it is difficult to tell which part in a work order is immobilizing the train.

In general, an ‘high risk’ part out of stock risks immobilizing a train.
We will estimate this cost at $10,000.
If a ‘low risk’ part is out of stock, it will cost $200.

$$\delta_{SOC} = 1 \; if \; S_t = 0$$
$$\delta_{SOC} = 0 \; if \; S_t > 0$$

$$IMc = \delta_{SOC} * CT * 10\,000 + (1 - CT) * 200$$

An operational reality says that if more than 20 trains are immobilized, the STM is no longer able to deliver the service it promised. I'd like my cost function to say that if more than 20 ‘high risk’ part number are out of order, the downtime cost is distorted by $25,000 for every unit of item out of order beyond that of 20 units.

$$IMc = \sum \delta_{SOC} * CT * 10\,000 + \max(\,0, (\sum \delta_{SOC} * CT) - 20\,) * 15\,000 + (\,1 - CT\,) * 200$$

Purchase Order cost :

In the ordering process, several processes are involved. By order of magnitude of the time each of these tasks takes, multiplied by the average hourly wage of a buyer, we can say that the cost of placing an order is $150.

$$\delta_{POC} = 1 \; if \; A_t > 0$$
$$\delta_{POC} = 0 \; if \; A_t = 0$$

$$PO_c = 150 * \delta_{POC}$$

If can order multiples items to a supplier with no additional cost. Meaning there is a dependency between costs function for series.

$$\delta_{FREE} = 1 \; if \; one \; part \; has \; already \; been \; ordered \; to \; the \; supplier$$
$$\delta_{FREE} = 0 \; otherwise$$
$$PO_c = 150 * \delta_{POC} * \delta_{FREE}$$

We could this probability approach to get rid of the dependency at first :

$$\mathrm{POc} = \mathbb{P}(\mathrm{i} \in \mathrm{B}) \cdot \frac{\mathrm{FC}}{\mathrm{N_{avg}}} + (1 - \mathbb{P}(\mathrm{i} \in \mathrm{B})) \cdot \mathrm{FC}$$

FC : fixed order cost ($)

$\mathbb{P}(\mathrm{i} \in \mathrm{B})$: Probability of the article i to be order in a bundle

N_{avg} : Average number of article in a bundle

Security Stock cost

We want to penalize, even slightly, the fact of consuming safety stock. Let’s say, $50 dollars if we consume the security stock at all.

$$SS_c = \max\left(0, \frac{SS - S_t}{SS}\right) * 50$$

Objective function

OBJECTIVE FUNCTION

Our problem consist of finding an optimal solution to the following problem:

$$\min_{\Pi} \sum_{t=0}^{T} \mathbb{E}[c(S_t, \underbrace{\Pi_t(S_t)}_{A_t})|S_0]$$

Solution (Backward Induction)

Backward Induction

- Starting from the **last time step**, the algorithm computes the **optimal action** for each state at each time step in backwards.
- At each step, it selects the action that **minimizes the expected cumulative cost** until the end of the horizon.
- Let **q_{t}(s, a)** be the expected cost until the end of the horizon when the agent takes action (a) in state (s) at time (t).

Bellman Update Equation

- For all state, action, time step tuple (s, a, t), define q_t(s, a) as

$$q_t(s,a) = c(s,a) + \sum_{s'} P(s'|s,a) \min_{a'} q_{t+1}(s',a')$$

$$\pi_t(s) = \arg\min_a q_t(s,a)$$

- **Boundary condition:** q_{**T+1**}(s, a) = 0 for all states and action pairs (s, a).

Computational Complexity is linear in T and |A|, and quadratic in |S|.

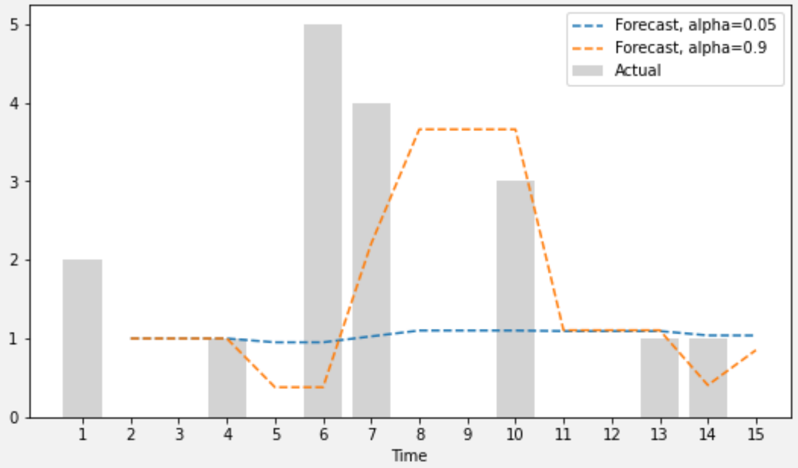
Demand Prediction (for all time steps)

As a separate approach, we can predict demand for the whole time-horizon by using a time series approach.

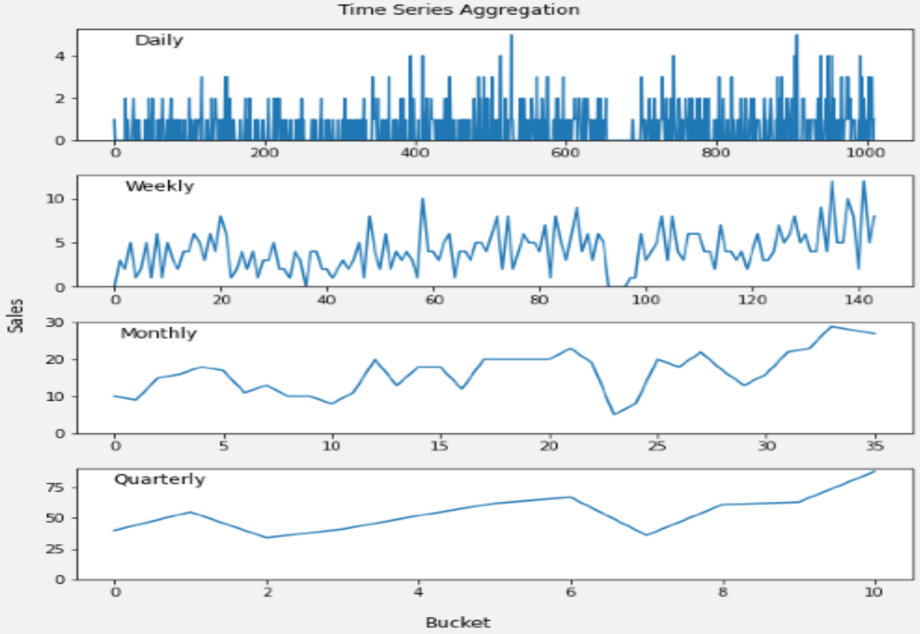
The literature propose such model

- Croston's
- TSB
- MAPA
- ADIDA
- Exponential smoothing

The backward induction can be implemented by assuming d_t is known for the whole time horizon.



Croston's



THANKS !