

Probability in Number Theory: a workshop on the distribution of L -functions and of multiplicative functions

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Titles & abstracts

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Large Deviations of the Riemann Zeta Function and Random Walks

Abstract. In these two talks, we will present a proof that the measures of level sets of the Riemann zeta function have Gaussian tail, up to a constant C , for values in suitable regimes. The lower bound, explained in the first talk, is unconditional, whereas the upper bound, proved in the second talk, necessitates the Riemann hypothesis for high enough values. As a corollary, we recover the best-known bounds on the moments on the critical line. The proof relies on the recursive scheme of prior work with Bourgade & Radziwill that is inspired by a random walk heuristic. The upper bound also combines ideas of Soundararajan and Harper. We will discuss possible improvements to the constant C as well as the connections with the Keating–Snaith Conjecture from Random Matrix Theory for the optimal constant.

This is joint work with Asher Roberts and Nathan Creighton.

Pierre-Alexandre Bazin[†] and Besfort Shala[‡]

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Modified Dirichlet Characters

Abstract. We report on recent progress towards understanding the growth of partial sums of modified Dirichlet characters. Addressing a conjecture of Klurman, Mangerel, Pohoata, and Teräväinen, our results improve upon several previous works for general modifications, and we establish a sharp result when the modifications are roots of unity. This is joint work arising from a collaboration during the thematic program.

Paul Bourgade

Courant Institute, NYU

Loop equations characterize random matrix statistics (joint work with Jiaoyang Huang)

Abstract. Loop equations are a hierarchy of identities relating correlation functions. I will explain how the universal local point processes of random matrix theory are uniquely characterized by their loop equation hierarchies. These hierarchies can be derived by integration by parts for many random matrix and random graph models. I will then discuss conjectural analogues for L -functions.

Hung Bui

University of Manchester

Weighted central limit theorem for central values of L -functions

Abstract. A classical result of Selberg says that $\log |\zeta(1/2 + it)|$ has a Gaussian limit distribution. We expect the same thing holds for $\log |L(1/2, \chi)|$ for χ being over the primitive Dirichlet characters modulo q , as q tends to infinity. Proving such a result remains completely out of reach, as it would imply 100% of these central L -values are non-zero, which is a well-known open conjecture. In this talk, I will describe how one can establish a weighted central limit theorem for the central values of some families of L -functions. This is based on joint work with Natalie Evans, Stephen Lester and Kyle Pratt, and with Alexandra Florea and Micah Milinovich.

Vorrapan Chandee

Kansas State University

On a large orthogonal family of L -functions

Abstract. We study a large orthogonal family of L -functions associated with holomorphic Hecke newforms of level q , averaged over $q \asymp Q$. I will talk about one level density and the n th centered moments of this family. Assuming GRH, the support of the Fourier transform of the test function on these statistics is extended to be roughly double the range from previous results. Moreover, we study one level density for large even and odd orthogonal families, with applications to non-vanishing at the critical point. The talk is based on my joint work with S. Baluyot and X. Li, X. Li and M. Milinovich, and X. Li and Y. Lee.

Cécile Dartyge

Université de Lorraine

Powerfree ellipsephic integers

Abstract. Let $b \in \mathbb{N}$, $b \geq 3$ and $\mathcal{D} \subsetneq \{0, \dots, b-1\}$. The ellipsephic integers, or integers with missing digits, are the integers with all their base- g digits in $\mathcal{D}$. In this talk we will present some results on the existence of powerfree ellipsephic integers in the case of subsets $\mathcal{D}$ of small cardinality.

This is joint work with Anne de Roton and Thomas Stoll.

Régis de La Bretèche

Université Paris Cité

The Central Limit Theorem for extended Rademacher random multiplicative random with polynomial phase

Abstract. We will present some news results on extended Rademacher random multiplicative functions with polynomial phase. This is joint work still in progress with Victor Wang and Max Xu.

Sary Drappeau

Université Clermont-Auvergne

Self-intersections points of expanding horocycles

Abstract. The expanding horocycle $H(y) = \{x + iy : 0 \leq x < 1\} \subset \mathrm{SL}(2, \mathbb{Z}) \backslash \mathbb{H}$ intersects itself at about y^{-2} points, as $y \rightarrow 0$. This talk will concern various aspects of the distribution of these points and the arcs they delimit, with a few results and many questions. This is joint work with Min Lee.

Kevin Ford

University of Illinois at Urbana-Champaign

Prime gaps, Hardy–Littlewood conjectures, the interval sieve and random models for primes

Abstract. We'll discuss connections between the distribution of primes in short intervals, gaps between consecutive primes, the Hardy–Littlewood k -point correlation conjectures, extremal behavior of interval sieves and random models for primes (Cramér, Granville, Banks–Ford–Tao).

Alexandra Florea

UC Irvine

Simultaneous non-vanishing of L-functions at the central point

Abstract. In this talk, I will focus on simultaneous non-vanishing results for Dirichlet L -functions at the central point $1/2$. Specifically, I will describe how to obtain a positive proportion of simultaneous non-vanishing result for four L -functions as we vary over characters χ modulo q , conditional on GRH. This is based on joint work with Hung Bui and Micah Milinovich.

Leo Goldmakher

Williams College

A converse to a theorem of Gauss on Gauss sums

Abstract. Gauss famously proved that the Gauss sum of any nontrivial character (mod p) has magnitude $\sqrt{p}$; more generally, the Fourier transform of any nontrivial character (mod p) has magnitude 1 at all nonzero inputs. We prove a result that is essentially a converse to this, for example: if f is an arbitrary function from $\mathbb{F}_p$ to $\{0, 1, -1\}$ that vanishes only at 0, and the Fourier transform of f has magnitude 1 somewhere, then f must be either the quadratic character (mod p) or its negative. One striking feature of our results is that they deduce multiplicative structure from a single analytic data point. We will also discuss several consequences of our rigidity theorem. Joint work with Jonathan Bober.

Ofir Gorodetsky

Technion – Israel Institute of Technology

The distribution of partial sums of the Steinhaus function

Abstract. The Steinhaus function is a random, completely multiplicative function on the integers, whose values on primes are i.i.d. random variables uniformly distributed on the complex unit circle. Its study is motivated by the study of “oscillatory” multiplicative functions such as Dirichlet characters.

We’ll describe recent joint work with Mo Dick Wong, where the limiting distribution of the partial sums of the Steinhaus function was determined. The limiting distribution is Gaussian with random variance; the variance is given by the total mass of a random measure. This measure is an instance of critical multiplicative chaos. We’ll explain the result and highlight the key ideas of the proof.

Adam Harper

University of Warwick

André Aisenstadt lectures

Abstract.

Lecture 1 (public lecture): Multiplicative functions and probability Many of the functions of greatest interest to analytic number theorists have a property called multiplicativity. Examples include the Mobius function, which is tied up with the Riemann Hypothesis; and Dirichlet characters, which are tied up with the distribution of sequences in arithmetic progressions. To guess how these functions might behave, it turns out to be fruitful to study probabilistic models. Recently, connections have been found with quite subtle probabilistic issues such as branching random walk and multiplicative chaos. It also turns out that one can (sometimes) use the probabilistic models not only to guess, but also to prove, results about the deterministic multiplicative functions that one started with. I will try to give a gentle introduction and overview of some of these issues.

Lecture 2: Finding the distribution of random multiplicative functions in short intervals Consider the sum $\sum_{x < n \leq x+y} f(n)$ of a random multiplicative function in a short interval (where $y = o(x)$ as x tends to infinity). Thanks to work of Chatterjee–Soundararajan and Soundararajan–Xu, it is known that these sums have a Gaussian limiting distribution when rescaled by their standard deviation, provided x/y is at least a certain power of $\log x$. On the other hand, work of Harper and of Caich implies that these sums will converge to zero when rescaled by their standard deviation, if y is “close” to x . I will report on joint work of myself, Soundararajan and Xu on this problem. We find that on the full range $y = o(x)$, the sums have a Gaussian limiting distribution when rescaled properly, but the correct scaling factor changes as y approaches x . In contrast, when $y \asymp x$ there is no rescaling under which the sums have a (non-degenerate) Gaussian limit.

Lecture 3: Lower bounds for low moments of character sums I will discuss the problem of bounding moments of sums $\sum_{n \leq x} \chi(n)$, where χ varies over all (non-principal) Dirichlet characters mod r . More specifically, I will be interested in obtaining lower bounds for the “low moments” (up to the second moment). In previous work, I proved upper bounds for these moments which match the predictions coming from random multiplicative functions. I will report on recent work and work

in progress obtaining matching lower bounds, and also describe an application to a non-vanishing problem.

Oleksiy Klurman

University of Bristol

Halász-type theorem for logarithmic means of multiplicative functions and its consequences

Abstract. A groundbreaking result of Halasz asserts that the partial sums of a bounded multiplicative function f exhibit cancellations unless f is “close” to n^{it} . I will discuss a new sharp theorem of this form for logarithmic means, explaining some somewhat surprising phenomena that arise. I will further present several applications of these results addressing questions on deterministic and random multiplicative functions.

Valeriya Kovaleva

CRM, Montreal

On integers divisible by a shifted prime in a given interval

Abstract. Let $H(x, y, z)$ be the number of integers with a divisor in a given interval $(y, z]$. Counting such integers is closely related to the multiplication table problem. In 2008, Ford found the order of magnitude of $H(x, y, z)$ in all ranges of interest. In this talk we consider a variant of this problem, where the divisors are restricted to shifted primes. We observe multiple phase transitions in the behaviour of the adjusted function as well as the anatomy of integers possessing shifted prime divisors in a prescribed interval depending on its logarithmic length. This is joint work with R. Abi Abdallah, J. Schlitt, and N. Tardy.

Emmanuel Kowalski

ETH, Zurich

Frobenius distribution and Kronecker measures

Abstract. The talk will introduce the notion of Kronecker measures, motivated by equidistribution theorems for Frobenius elements, and illustrate it with various examples and open problems.

Youness Lamzouri

Université de Lorraine

Real zeros of $L'(s, \chi_d)$ and the Baker–Montgomery conjecture

Abstract. In 1990, R. C. Baker and H. L. Montgomery conjectured that for almost all fundamental discriminants d , the derivative of the Dirichlet L -function associated to the quadratic character modulo d has around $\log \log |d|$ real zeros on the interval $[1/2, 1]$. Baker and Montgomery’s motivation in studying these zeros stems from their connection to real zeros of Fekete polynomials and to sign

changes of real character sums. In this talk I will present recent work that settles this conjecture (up to a small logarithmic factor of $\log \log \log |d|$). This is based on a joint work with Oleksiy Klurman and Marc Munsch for the lower bound, and a more recent work joint with Kunjakanan Nath for the upper bound.

Junxian Li

UC Davis

Moments of twists of $\mathrm{GL}(3)$ L -functions

Abstract. Asymptotic formulas for twists of $\mathrm{GL}(3)$ L -functions have previously been established for factorizable moduli. In this talk, we discuss an asymptotic formula for $\mathrm{GL}(3)$ L -functions twisted by Dirichlet characters with moduli averaged over all $q < Q$. The problem is closely connected to a $\mathrm{GL}(3) \times \mathrm{GL}(2)$ shifted convolution problem, which we resolve using two intertwined applications of different types of the delta symbol method. This is based on joint work with Valentin Blomer.

James Maynard

University of Oxford

Exponential sums over primes

Abstract. A classical result of Vinogradov shows that the exponential sum over primes

$$\sum_{p \leq x} e(\alpha p)$$

is bounded by $x/q^{1/2} + x^{4/5}$, up to an $x^{o(1)}$ factor, whenever α has the Diophantine approximation $a/q + O(1/q^2)$ for some $q < x^{1/2}$. This has resisted improvements for the past 80 years, beyond refinements to the $x^{o(1)}$.

The $x/q^{1/2}$ term cannot be improved without a breakthrough on our understanding of Siegel zeros. I'll discuss how new methods allow us to improve the $x^{4/5}$ term to $x^{19/24}$, which in turn should have various applications to additive problems related to the primes.

Joël Rivat

Université d'Aix-Marseille

Prime numbers with an almost prime reverse

Abstract. Given an integer $b \geq 2$, we define the *reverse* in base b of an integer $n \geq 0$ to be the integer obtained by reversing the digits of n . The existence of infinitely many prime numbers whose reverse is also prime is an open problem which seems at least as difficult as the twin prime conjecture. In a joint work with Cécile Dartyge and Cathy Swaenepoel, we show that there are infinitely many prime numbers with an almost prime reverse. More precisely, we show that there exist an explicit $\Omega_b \in \mathbf{N}$ and $c_b > 0$ such that, for at least $c_b b^\lambda \lambda^{-2}$ primes $p \in [b^{\lambda-1}, b^\lambda]$, the reverse of p has at most Ω_b prime factors. The proof is based on sieve methods and a result we obtain in the spirit of the Bombieri–Vinogradov theorem concerning the distribution in arithmetic progressions of the reverse of prime numbers.

Brad Rodgers

Queen's University

An asymptotic for the fourth moment of the Hurwitz zeta function

Abstract. In this talk I will discuss an asymptotic formula for the fourth power moment of the Hurwitz zeta function $\zeta(1/2 + it, \alpha)$, for irrational α which are sufficiently poorly approximable. The result is consistent with a conjectured Gaussian distribution for the Hurwitz zeta function for such α . I hope to outline a proof of this result as well as explain motivations for the problem coming from a more general context. This is joint work in progress with W. Heap and A. Sahay.

Fernando Shao

University of Kentucky

Linear equations in Piatetski–Shapiro primes

Abstract. We establish decorrelation estimates between the Piatetski–Shapiro primes, primes of the form $\lfloor n^c \rfloor$, and arbitrary nilsequences, where $c > 1$ is sufficiently close to 1. This extends earlier works which treated linear or polynomial exponential phase functions and provides the first higher-order uniformity result for Piatetski–Shapiro primes. As a consequence, we obtain an asymptotic formula for linear equations in Piatetski–Shapiro primes, thereby generalizing the Green–Tao theorem on linear equations in primes to this sparse setting. This is joint work with Yu-Chen Sun.

Gérald Tenenbaum

Université de Lorraine

The Erdős–Hooley Delta-function: a survey

Abstract. Defined by Erdős and further developed by Hooley, the Delta-function reflects the concentration of the numbers $\log d$ when d runs through the divisors of a natural integer n . Despite many efforts made in recent years and significant progress achieved, its average and normal orders have not yet been fully elucidated. The aim of this lecture is to describe the various approaches that have been implemented to tackle this function and to provide an overview of old and new available results.

Joni Teräväinen

University of Cambridge

Linnik's problem for the Möbius function

Abstract. In 1944, Linnik showed that the least prime in an arithmetic progression is bounded polynomially in terms of the modulus of the progression. Since then, many works have focused on improving the exponent in the polynomial dependence. In this talk we consider an analogue of Linnik's problem for the Möbius function and prove that for this variant the exponent can be taken to be 2. This is based on joint work with Kaisa Matomäki.

Mo Dick Wong

Hong Kong University

Universality of critical multiplicative chaos

Abstract. The problem of universality in multiplicative chaos asks whether the limiting random measure depends only on the underlying log-correlated field, rather than on the particular regularisation through which the measure is constructed. This question arises naturally in recent joint work with Ofir Gorodetsky, where we need to show that different approximations of random Euler products lead to the same chaos measure in order to apply a martingale CLT to the study of partial sums of a Steinhaus multiplicative function. A key technical challenge at criticality is the lack of sufficient integrability to apply a direct second moment argument. In this talk, I will explain a modified approach that avoids the use of barrier events, relying instead on change-of-measure and coupling arguments.

Max Xu

Courant Institute, NYU

Escaping Chaos

Abstract. In this talk, I will summarize and explain some new results and some very new results in the study of chaotic behaviors of RMF where a phase transition phenomenon emerges.

Asif Zaman

University of Toronto

Effective Brauer–Siegel theorems for Artin L -functions

Abstract. Given a number field K , in a now classic work, Stark pinpointed the possible source of a so-called Landau–Siegel zero of the Dedekind zeta function $\zeta_K(s)$ and used this to give effective upper and lower bounds on the residue of $\zeta_K(s)$ at $s = 1$.

I will discuss an extension of Stark’s work to give effective upper and lower bounds for the leading term of the Laurent expansion of general Artin L -functions at $s = 1$ that are, up to the value of implied constants, as strong as could reasonably be expected given current progress toward the generalized Riemann hypothesis. The bounds are completely unconditional and rely on no unproven hypotheses about Artin L -functions. Strengthened conditional bounds also lead to further questions on the distribution of these values.

This is joint work with Peter Cho and Robert Lemke Oliver.
