Paul Jacob Koosis, 2003, Montréal

Paul Jacob Koosis (1929–2025): A Life Devoted to Mathematics and Culture

Paul Koosis was a brilliant, original, and deeply cultivated mathematician whose work shaped modern harmonic and complex analysis. His career was marked not by conventional paths, but by a passionate, lifelong engagement with ideas, books, and people. He was a teacher, a scholar, a writer, and, above all, a mathematician in the most classical sense.

Koosis was born in 1929 in San Francisco and completed his Ph.D. in 1954 at Berkeley under the direction of John Kelley, with a dissertation on Fourier coefficients of absolutely continuous measures. His early career brought him to the Courant Institute, where he remained until 1957 as a temporary member, supported by the NSF and the U.S. Office of Naval Research, working under the guidance of Louis Nirenberg. During this period, he briefly held a faculty position at Fordham University. However, this dual affiliation led to his dismissal from Courant, as it was seen as a conflict of commitment.

In 1957-58, supported by a Fulbright Fellowship, Koosis spent a year in Montpellier, France, where he met Yitzhak Katznelson and witnessed the blossoming of his thesis work. He also began a lifelong friendship with Jean-Pierre Kahane, a remarkable mathematician and political activist known for his involvement in the French Communist Party, who was a professor in Montpellier at the time. Koosis then held a teaching position in Paris, below his formal qualifications, but it was there that he initiated a new seminar in analysis at Orsay. The seminar flourished and later specialized in harmonic analysis. Paul developed a deep love for France, the French language, and French mathematics, especially the work of Beurling and Malliavin, which became his lifelong fascination.

Starting in the early 1960s, Paul joined UCLA, where he formed a close collegial bond with John Garnett. Their research began in Banach algebras, but with Paul's background and originality, they soon made substantial progress in this and adjacent areas of analysis. It was at UCLA that he played a pivotal role in the career of Peter Jones, who had initially enrolled in a real analysis course on a whim. Paul recognized his exceptional talent and, when Jones encountered bureaucratic issues, he intervened to secure his place at UCLA. Around 1965–66, when the department moved into a new wing, Paul was given an office far from everyone else, reportedly because he was building and playing a harpsichord in his office. Paul was truly a singular figure in the department. His harpsichord, his political convictions, his habit of speaking first in French, his uncompromising mathematical taste, and his daily eccentricities set him apart in a rapidly growing and ambitious environment. One Saturday, he and his second wife inadvertently broke the only Xerox machine in the department while attempting to use it to gather evidence that grocery stores in poor neighborhoods charged higher prices than those in affluent areas. He was unpredictable and fascinating. On one occasion, he went on strike, believing that a nearby small experimental nuclear reactor from the engineering school had made the building radioactive (it had not). He might or might not attend a seminar, could vanish for weeks, and then unexpectedly call late at night. And yet, he was utterly brilliant, deeply insightful in hard analysis, generous with his time and ideas, and unwavering in his commitment to intellectual integrity.

In the academic year 1967–68, he was at Imperial College in London.

Lennart Carleson, always supportive of Paul's brilliance, arranged for him to spend two consecutive academic years at the Mittag-Leffler Institute, beginning in 1977. This period of staying led to a major paper published in *Acta Mathematica* on the Beurling–Malliavin theorem, at a time when Beurling was still present. Paul extended his stay and taught at the Royal Institute of Technology. Like many foreign visitors to Sweden at the time, he also traveled to the former USSR, especially to Leningrad (currently, Saint Petersburg), which hosted a prominent group of harmonic analysts. There, in particular, he developed a close and enduring friendship with Victor Petrovich Havin, one that would later result in Havin spending a semester at McGill University in the 1990s.

Victor Petrovich Havin, Paul Koosis
An enduring friendship

In the 1980s, Paul followed his heart to Montreal and the Laurentians, where he found himself most productive. He joined McGill University, initially in a modest position, but his impact was anything but modest. He attracted and mentored a number of outstanding students and collaborators and became a central figure in Canada's classical analysis community. From 1989 to 1998, he served as professor in the Department of Mathematics with a reduced teaching load. In 2009, he was named Professor Emeritus.

Koosis's work focused on classical analysis: Hardy spaces, Fourier analysis, and especially the theory of Beurling and Malliavin, which became central to his life. His books *Introduction to Hp Spaces*, *The Logarithmic Integral* (Volumes I and II), and *Leçons sur le théorème de Beurling et Malliavin* (published in French by the CRM) are regarded as essential references in these areas. He was deeply passionate about the French language and declined offers to have his last monograph translated or published in English. These texts do not merely convey results; they breathe life into ideas through clarity, precision, and a love of exposition. His work on the Beurling-Malliavin theory (a foundational result concerning the completeness of exponential systems, with roots in the work of Paley, Wiener, Levinson, and Schwartz) was not only technically deep but expressed with great literary care.

In October 2003, a conference was held in Montreal in his honour, co-organized by the CRM and several Quebec universities, including McGill. The event brought together a generation of very prominent figures (D. Drasin, J. Garnett, V. Havin, W. Hayman, P. Jones, J.-P. Kahane, F. Nazarov, J. Nirenberg, H. Petersen, T. Ransford, M. Sodin, S. Treil, A. Volberg, M. Wilson, etc.). The success of this conference illustrated not only the vitality of classical analysis but also Koosis's foundational place within it.
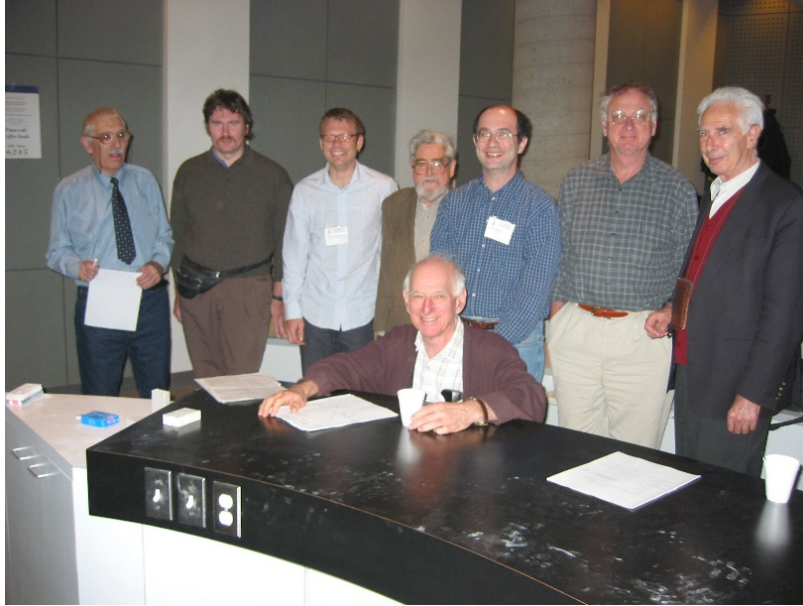
The 2003-CRM Conference
V. Havin, F. Nazarov, H. Petersen, J. Nirenberg, M. Sodin, J. Garnett, J.-P. Kahane
P. Koosis

Paul was also a deeply respected and beloved teacher. For several decades, he taught courses in analysis, and his students consistently admired both the clarity of his lectures and the depth of his commitment. Even well into his seventies, he remained intellectually vibrant, actively working, teaching, and contributing to the field. He paid meticulous attention to student homework, very often returning it with detailed annotations and personal notes. Beyond correcting mathematics, he took great care to teach his students how to write with precision and elegance, emphasizing style and what he called "mathematical grammar."
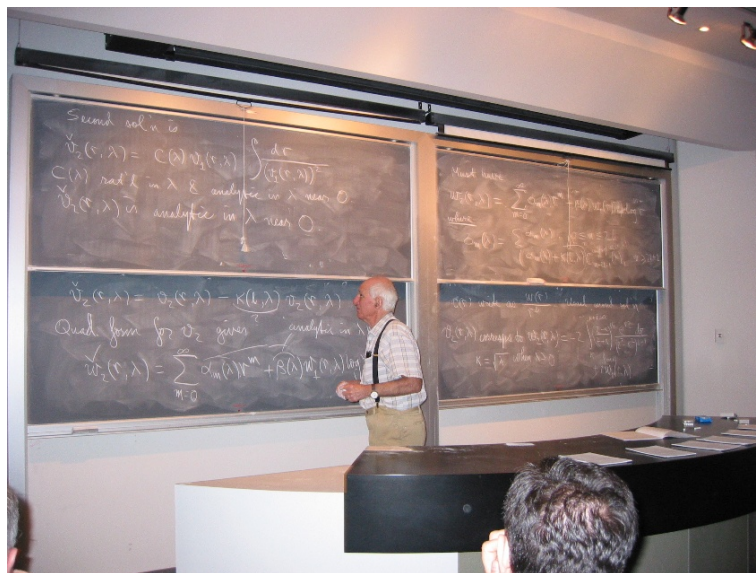
Paul Koosis, at CRM, 2003

Perhaps most memorable was his unique way of responding to mathematical questions. Many of us recall with awe the experience of asking Paul something we were struggling with. He would begin humbly, saying he wasn't sure of the answer, but just a few days later, you would receive several handwritten pages: beautifully crafted, impeccably logical, and rendered in his famously elegant script. His handwritten mathematics was a true form of art. There are numerous stories of this kind, shared across generations and disciplines, testifying to his generosity, brilliance, and profound influence as a mentor.

In addition to being a gifted mathematician, Paul Koosis was a man of wide and passionate intellectual interests. He played the harpsichord, read voraciously, and loved authors such as Gabrielle Roy, Christa Wolf, Shakespeare, Goethe, and Heine. Paul was also a remarkable humanitarian. His second volume of *The Logarithmic Integral* begins with a moving memorial: the names and ages of the 14 young women murdered at École Polytechnique in Montreal on December 6, 1989; a gesture of empathy and remembrance that speaks volumes about the man behind the mathematics.

Remember

Geneviève Bergeron, age 21,

Hélène Colgan, age 23,

Nathalie Croteau, age 23,

Barbara Daigneault, age 22,

Anne-Marie Edward, age 21,

Maud Haviernick, age 29,

Barbara Maria Klucznik-Widajewicz, age 31,

Maryse Laganière, age 25,

Maryse Leclair, age 23,

Anne-Marie Lemay, age 22,

Sonia Pelletier, age 28,

Michèle Richard, age 21,

Annie St-Arneault, age 23,

Annie Turcotte, age 20;

murdered in the Montreal Ecole Polytechnique on December 6, 1989.

Logarithmic Integral, II, Dedication Page

I conclude with two personal encounters with Paul that I will always cherish. For many years, Paul lived in Outremont with *Suzanne Gervais*, one of the pioneers of Canadian animated cinema. In the last decade of Paul's life, Suzanne was diagnosed with Alzheimer's and was eventually moved to a specialized care facility. Every single day, Paul would take public transport across Montreal and then walk two kilometers to visit her, staying as long as the staff would allow. Once, I asked Paul why he continued to do this, knowing that Suzanne no longer recognized him. He gently replied, "*But, I recognize her.*"

Suzanne Gervais and Paul Koosis, at CRM, 2003

Second, around the year 2000, I learned that he was working on a challenging approximation problem. Over the years, I asked about his progress several times, and he always replied, "Not yet finished." Then, twenty years later during a phone call in Covid era, he told me it was nearly done. I invited him to submit the work to a volume I was editing for the Fields Institute. He agreed and a few weeks later, I received a 142-page handwritten manuscript in the mail. It was a masterpiece, the culmination of decades of reflection and insight, written at the age of 95. Paul never learned to type or use the internet, not out of inability, but by choice. He remained committed to his own way of working, relying on handwritten notes and personal correspondence. Through this personal approach, he made deep and lasting contributions to mathematics, leaving behind a remarkable and elegant legacy. Later in 2023, I met Paul in his Outremont apartment to give him printed copies of the paper. He showed me a large stack of earlier drafts and revisions. What I had received was the final one, the culmination of tireless dedication and mathematical craftsmanship.

Paul Koosis, Outremont, 2023

Further information about the technical aspects of Koosis' mathematical work can be found in an article by J.-P. Kahane in Issue 100 of the *Gazette* of the French Mathematical Society. That article also served as a source for some of the material included in this note. I am also grateful to several of Paul's longtime friends who helped me gather information; my special thanks go to Michael Barr, David Drasin and John Garnett.

Paul Koosis was a unique intellectual presence, a man who lived not for position or prestige, but for beauty, truth, dignity and understanding. He carried this quiet integrity with him throughout his life, shaping every lecture, every page, every conversation. His legacy is profound, not only in the deep theorems and influential books he leaves behind, but also in the minds he inspired and the lives he gently transformed. We remember him with deep respect and affection. Our thoughts are with his family, his colleagues, and the many mathematicians across the world who had the good fortune to know him, learn from him, and admire the luminous clarity of his spirit.

Javad Mashreghi

Université Laval