Black holes and warp travel

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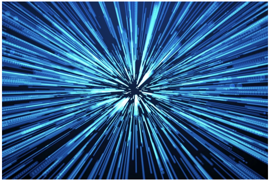
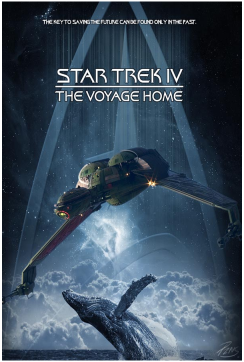
Analyse de singularités géométriques



What's your favorite Star Trek? Please go to menti.com and enter code 5404 0478.

- Movie or series or character(s)?
- Enter never seen if you have never seen Star Trek. (really???)



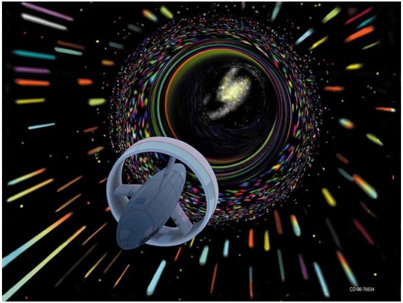
Hyperbolic orbifold surface

- Hyperbolic upper half plane
  $\mathbb{H} = \{z = x + iy | x \in \mathbb{R}, \quad y \geq 0, \quad i^2 = -1\}$.
- Linear fractional transformations: $z \mapsto \frac{az+b}{cz+d}$ for $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.
- Equivalence: $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cong \begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix}$, assume $|\det| = 1$ gives the group $\mathrm{PSL}_2(\mathbb{R})$ of orientation preserving isometries acting on $\mathbb{H}$.
- Let Γ be a discrete subgroup.
- Quotient space $X = \Gamma \backslash \mathbb{H}$ is our hyperbolic orbifold surface.
- Assume X is compact.

Engage!

Theorem (Selberg)

If spacetime is a hyperbolic orbifold surface that is compact and smooth, then warp travel is possible.



Good things come in 3s

- Since X is compact, the group Γ may have three types of elements.
- The identity element.
- Hyperbolic elements.
- Elliptic elements.

Hyperbolic elements and closed geodesics

- $P \in \Gamma$ is *hyperbolic* if it is $\mathrm{PSL}_2(\mathbb{R})$-conjugate to

$$\begin{pmatrix} a(P) & 0 \\ 0 & a(P)^{-1} \end{pmatrix}, \quad 1 < a(P).$$

- P gives rise to a closed geodesic in $\Gamma \backslash \mathbb{H}$ with length $\ell_P = \log\left(|a(P)|^2\right)$.
- Largest k such that $P = P_0^k$ for some $P_0 \in \Gamma$; if $k = 1$, $P = P_0$ is a *primitive hyperbolic element*.
- Set of Γ-conjugacy classes of all hyperbolic elements, respectively primitive hyperbolic elements, by $\{\mathcal{P}\}$, respectively $\{\mathcal{P}\}_p$.

Elliptic elements

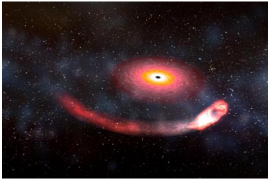
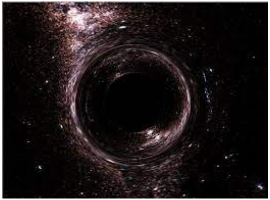
- $R \in \Gamma$, is *elliptic* if it is of finite order (and not the identity).
- Any cyclic subgroup, $\mathcal{R}$ of finite order in Γ is generated by a *primitive elliptic element* R_0 of order $m_{\mathcal{R}} \in \mathbb{N}$.
- This element be chosen in $\mathrm{PSL}_2(\mathbb{R})$ to be conjugate to

$$\begin{pmatrix} \cos(\pi/m_{\mathcal{R}}) & -\sin(\pi/m_{\mathcal{R}}) \\ \sin(\pi/m_{\mathcal{R}}) & \cos(\pi/m_{\mathcal{R}}) \end{pmatrix}.$$

- The angle, $\theta_{\mathcal{R}} = \pi/m_{\mathcal{R}}$, is the smallest positive angle among all such angles determined by the elements of the group generated by R_0.
- The set of all primitive elliptic elements of Γ is $\{\mathcal{R}\}_p$.

Elliptic elements, conical singularities, and black holes

- If Γ has elliptic elements, then X has conical singularities.
- Cosmic strings and Schwarzschild black holes.
- These could make warp travel impossible.

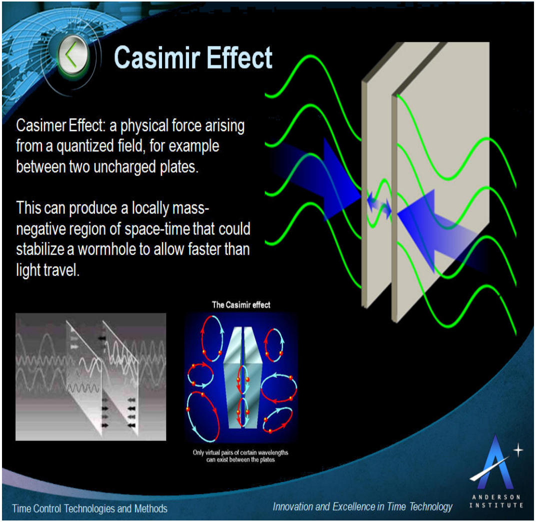
Eigenvalues and spectrum

- X inherits the hyperbolic Riemannian metric.
- Laplace eigenfunctions $u \in H^2(X)$ and eigenvalues $\lambda \in \mathbb{C}$

$$\Delta u(x,y) = \lambda u(x,y), \quad \Delta = -y^2(\partial_x^2 + \partial_y^2),$$

  such that $u(\gamma z) = u(z)$ for all $z \in \mathbb{H}$, $\gamma \in \Gamma$.
- $0 = \lambda_0 < \lambda_1 \leq \cdots \nearrow \infty$, with λ_n on the order of n.
- The set of all eigenvalues is the spectrum.

Casimir energy



Mathematically...

- Casimir energy is calculated through the spectral zeta function

$$\zeta(s) := \sum_{n \geq 1} \lambda_n^{-s},$$

- Casimir energy is $\zeta(-1/2)$.
- What's wrong with this? (Please answer in menti.com 5404 0478)

Reformulation of the spectral zeta function

Theorem (K. Fedosova, JR & G. Zhang 2024)

For $s \in \mathbb{C} \setminus \mathbb{N}_{\geq 1}$

$$\zeta(s) = \frac{\mathrm{vol}(\Gamma \backslash \mathbb{H})}{8(s-1)} 4^s \pi^{s-1} \Gamma(2-s) \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{(n+1)(-1)^k}{2^{n+3/2+s}} \binom{n}{k}$$

$$\times \frac{\boldsymbol{K}_{3/2-s}(\pi + \pi k)}{(1+k)^{3/2-s}}$$

$$+ \dots$$

Continued

Theorem (continued)

$$+\frac{(4\pi)^{-1/2}}{\Gamma(s)}\sum_{\{\mathcal{P}\}_p}\sum_{n=1}^{\infty}(\ell_\gamma/n)^{1/2}\operatorname{csch}(\tfrac{n\ell_\gamma}{2})(n\ell_\gamma)^s K_{1/2-s}(\tfrac{n\ell_\gamma}{2}).$$

$$+\sum_{\{\mathcal{R}\}_p}\sum_{\ell=1}^{m_{\mathcal{R}}-1}\frac{2^{s-5/2}\sqrt{\pi}\Gamma\left(1-s\right)}{m_{\mathcal{R}}\sin(\frac{\pi\ell}{m_{\mathcal{R}}})}\sum_{n=0}^{\infty}2^{-n-1}\sum_{k=0}^{n}\binom{n}{k}$$

$$\times\frac{(-1)^k \boldsymbol{K}_{-s+1/2}(\frac{\pi\ell}{m_{\mathcal{R}}}+\pi k)}{(\frac{\pi\ell}{m_{\mathcal{R}}}+\pi k)^{-s+1/2}}$$

Consequences

- ζ extends meromorphically to $\mathbb{C}$ with a single simple pole at 1. (Removable singularities at $\mathbb{N}_{\geq 2}$.)
- The Casimir energy is $\zeta(-1/2)$ is well defined and equals

$$\frac{\text{vol}(\Gamma\backslash\mathbb{H})}{\pi}\sum_{n=0}^{\infty}\frac{n+1}{2^{n+6}}\sum_{k=0}^{n}(-1)^{k+1}\binom{n}{k}\frac{\boldsymbol{K}_2\left[\pi(1+k)\right]}{(1+k)^2}$$

$$-\frac{1}{4\pi}\sum_{\{\mathcal{P}\}_p}\sum_{n=1}^{\infty}\tfrac{1}{n}\,\text{csch}\left(\tfrac{n\ell_\gamma}{2}\right)K_1\left(\tfrac{n\ell_\gamma}{2}\right)$$

$$+\sum_{\{\mathcal{R}\}_p}\sum_{\ell=1}^{m_{\mathcal{R}}-1}\frac{1}{8m_{\mathcal{R}}\sin\left(\frac{\pi\ell}{m_{\mathcal{R}}}\right)}\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}\sum_{k=0}^{n}(-1)^k\binom{n}{k}\frac{\boldsymbol{K}_1\left[\left(k+\frac{\ell}{m_{\mathcal{R}}}\right)\pi\right]}{k+\frac{\ell}{m_{\mathcal{R}}}}$$

K and $\boldsymbol{K}$

Definition

K_j is the modified Bessel function of the second kind of order j. $\boldsymbol{K}_j$ is the j-th Struve function of the second kind,

$$\boldsymbol{K}_j(z) = \boldsymbol{H}_j(z) - Y_j(z).$$

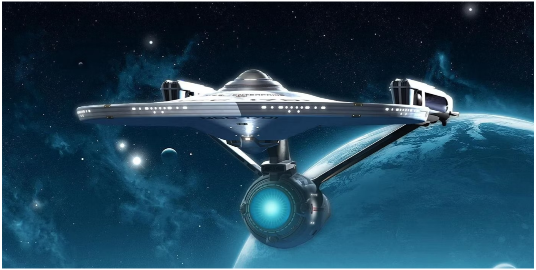
Here, $\boldsymbol{H}_j$ is the j-th Struve function of the first kind, and Y_j the Bessel function of the second kind, a.k.a. the Weber Bessel function (for various expressions of these see Watson, DLMF, NIST).

Why is it useful and how does one prove it?

- This expression converges rapidly (exponentially).
- The Struve functions can be evaluated with systems of computer algebra to arbitrary precision.
- We prove the formula using the Selberg trace formula, together with new calculations of the elliptic orbital integrals ($\heartsuit$ Ksenia).

Can the enterprise engage?

- For warp travel one needs Casimir energy to be negative.
- The first and second terms (identity and hyperbolic) are negative.
- The elliptic term is positive.
- Which sign dominates?



Triangle groups

- Fix $p, q, r \in \mathbb{N}$ with $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} < 1$.
- Define a (p, q, r)-*triangle group* as in (Definition 10.6.3, Beardon), denoted $\Gamma(p, q, r)$.
- $\Gamma(p, q, r)$ is a discrete subgroup of $\mathrm{PSL}_2(\mathbb{R})$.
- The area (volume) of $\Gamma(p, q, r)\backslash\mathbb{H}$ is

$$2\pi\left(1 - \left(\tfrac{1}{p} + \tfrac{1}{q} + \tfrac{1}{r}\right)\right). \tag{1}$$

The 2-3-7 orbifold surface as our spacetime

Specify to $\Gamma(2,3,7)\backslash\mathbb{H}$. In general, (and on this surface) for $s \neq 1$

$$\zeta(s) = \frac{\mathrm{vol}(\Gamma\backslash\mathbb{H})}{8(s-1)} \int_{\mathbb{R}} (\tfrac{1}{4} + r^2)^{1-s} \operatorname{sech}^2(\pi r) dr$$
$$+ \frac{(4\pi)^{-1/2}}{\Gamma(s)} \sum_{\{\mathcal{P}\}_p} \sum_{n=1}^{\infty} (\ell_\gamma/n)^{1/2} \operatorname{csch}(\tfrac{n\ell_\gamma}{2}) (n\ell_\gamma)^s K_{1/2-s}(\tfrac{n\ell_\gamma}{2})$$
$$+ \sum_{\{\mathcal{R}\}_p} \sum_{\ell=1}^{m_{\mathcal{R}}-1} \frac{1}{2m_{\mathcal{R}} \sin(\frac{\pi\ell}{m_{\mathcal{R}}})} \int_{\mathbb{R}} \frac{e^{-2r\frac{\pi\ell}{m_{\mathcal{R}}}}}{1+e^{-2\pi r}} (\tfrac{1}{4} + r^2)^{-s} dr.$$

Elliptic energy

Lemma

The contribution of elliptic elements to the Casimir energy is equal to

$$\sum_{\{\mathcal{R}\}_p} \sum_{\ell=1}^{m_{\mathcal{R}}-1} \frac{1}{8 m_{\mathcal{R}} \sin(\frac{\pi \ell}{m_{\mathcal{R}}})} \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} \sum_{k=0}^{n} (-1)^k \binom{n}{k} \frac{\boldsymbol{K}_1 \left[(k + \frac{\ell}{m_{\mathcal{R}}})\pi \right]}{k + \frac{\ell}{m_{\mathcal{R}}}}.$$

Elliptic energy estimate

Lemma

The elliptic contribution to the Casimir energy of $\Gamma(2, 3, 7) \setminus \mathbb{H}$ *rounded to six decimal places is equal to 0.875676.*

Identity energy

Lemma

We have

$$\frac{\mathrm{vol}(\Gamma\backslash\mathbb{H})}{8(s-1)}\int_{\mathbb{R}}(\tfrac{1}{4}+r^2)^{1-s}\,\mathrm{sech}^2(\pi r)dr\Big|_{s=-1/2}$$

$$=-\frac{\mathrm{vol}(\Gamma\backslash\mathbb{H})}{\pi}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(n+1)(-1)^k}{2^{n+6}}\binom{n}{k}\frac{\boldsymbol{K}_2(\pi+\pi k)}{(1+k)^2}.$$

Identity energy estimate

Lemma

The identity contribution to the Casimir energy is at least

$$-\frac{2\,\mathrm{vol}(\Gamma\backslash\mathbb{H})}{45\pi}.$$

Observe:

$$\frac{\mathrm{vol}(\Gamma\backslash\mathbb{H})}{8(s-1)}\int_{-\infty}^{\infty}(\tfrac{1}{4}+r^2)^{1-s}\,\mathrm{sech}^2(\pi r)dr\Big|_{s=-1/2}$$
$$=-\frac{\mathrm{vol}(\Gamma\backslash\mathbb{H})}{12}\int_{-\infty}^{\infty}(\tfrac{1}{4}+r^2)^{\frac{3}{2}}\,\mathrm{sech}^2(\pi r)dr.$$

Identity energy estimate continued

$$-\frac{\mathrm{vol}(\Gamma\backslash\mathbb{H})}{12}\int_{-\infty}^{\infty}(\tfrac{1}{4}+r^2)^{\frac{3}{2}}\operatorname{sech}^2(\pi r)dr \geq$$
$$-\frac{\mathrm{vol}(\Gamma\backslash\mathbb{H})}{96}\int_{-\infty}^{\infty}(1+4r^2)^2\operatorname{sech}^2(\pi r)dr.$$

By Gradshteyn & Rizhik 3.527.3.12

$$\int_0^\infty \frac{x^{b-1}}{\cosh^2 x}dx = 2^{2-b}(1-2^{2-b})\Gamma(b)\zeta_R(b-1), \quad \mathrm{Re}(b)>0, \quad b\neq 2,$$
$$\Longrightarrow$$
$$\int_{-\infty}^{\infty}(4r^2+1)^2\operatorname{sech}^2(\pi r)dr = 2\int_0^\infty(16r^4+8r^2+1)\operatorname{sech}^2(\pi r)dr =$$
$$2\left(\int_0^\infty\left(\frac{16s^4}{\pi^4}\operatorname{sech}^2(s)+\frac{8s^2}{\pi^2}\operatorname{sech}^2(s)+\operatorname{sech}^2(s)\right)\frac{ds}{\pi}\right) =$$
$$2\left(\frac{16}{\pi^5}2^{2-5}(1-2^{2-5})\Gamma(5)\zeta(4)+\frac{8}{\pi^3}2^{2-3}(1-2^{2-3})\Gamma(3)\zeta(2)+\frac{1}{\pi}\right) =$$
$$2\left(\frac{7}{15\pi}+\frac{2}{3\pi}+\frac{1}{\pi}\right) = \frac{64}{15\pi}. \implies \text{lower bound}$$

$$-\frac{\mathrm{vol}(\Gamma\backslash\mathbb{H})}{96}\cdot\frac{64}{15\pi} = -\frac{2\,\mathrm{vol}(\Gamma\backslash\mathbb{H})}{45\pi} > -0.00211640.$$

Hyperbolic contribution

$$\frac{(4\pi)^{-1/2}}{\Gamma(-1/2)} \sum_{\{\mathcal{P}\}_p} \sum_{n=1}^{\infty} (\ell_\gamma/n)^{1/2} \operatorname{csch}(\tfrac{n\ell_\gamma}{2})(n\ell_\gamma)^{-1/2} K_1(\tfrac{n\ell_\gamma}{2})$$

$$= -\frac{1}{4\pi} \sum_{\{\mathcal{P}\}_p} \sum_{n=1}^{\infty} (\ell_\gamma/n)^{1/2} \operatorname{csch}(\tfrac{n\ell_\gamma}{2})(n\ell_\gamma)^{-1/2} K_1(\tfrac{n\ell_\gamma}{2})$$

$$= -\frac{1}{4\pi} \sum_{\{\mathcal{P}\}_p} \sum_{n=1}^{\infty} \tfrac{1}{n} \operatorname{csch}(\tfrac{n\ell_\gamma}{2}) K_1(\tfrac{n\ell_\gamma}{2}).$$

Hyperbolic elements and closed geodesics

The rotations of order 3 and 7 that generate the triangle group $\Gamma(2,3,7)$ are represented as elements of $\mathrm{PSL}_2(\mathbb{R})$ using

$$b = \frac{1}{3}\left(\sqrt{3(\cot^2(\frac{\pi}{7}) - 3)} + \sqrt{3}\cot(\frac{\pi}{7})\right),$$

$$A = \begin{pmatrix} \cos(\frac{\pi}{3}) & \sin(\frac{\pi}{3}) \\ -\sin(\frac{\pi}{3}) & \cos(\frac{\pi}{3}) \end{pmatrix}, \quad B = \begin{pmatrix} \cos\left(\frac{\pi}{7}\right) & b\sin\left(\frac{\pi}{7}\right) \\ -b^{-1}\sin\left(\frac{\pi}{7}\right) & \cos\left(\frac{\pi}{7}\right) \end{pmatrix},$$

$$R = A^{-1}B, \quad L = B.$$

Each hyperbolic element γ is represented as a matrix M_γ that is a product of R and L. The length of the corresponding closed geodesic is

$$2\cosh^{-1}|\mathrm{Tr}(M_\gamma))/2|.$$

Multiplicities

- Let γ be a primitive hyperbolic element and denote the length of the corresponding closed geodesic by ℓ_γ.
- γ^{-1} is also a hyperbolic element with $\ell_{\gamma^{-1}} = \ell_\gamma$.
- Changing the R's and L's in the representation of γ, one obtains a hyperbolic element γ^* with $\ell_{\gamma^*} = \ell_\gamma$.
- $s(\gamma)$ is the number of distinct conjugacy classes among $\{\gamma\}, \{\gamma^{-1}\}$, $\{\gamma^*\}$ and $\{(\gamma^*)^{-1}\}$.

Computing lengths ♡ Vogeler

$$A(\gamma) := -\frac{1}{4\pi}\sum_{n=1}^{\infty}\frac{s(\gamma)}{n}\operatorname{csch}\left(\frac{n\ell_\gamma}{2}\right)K_1\left(\frac{n\ell_\gamma}{2}\right).$$

$\ell_\gamma \approx$	$s(\gamma)$	γ	$A(\gamma) \approx$
0.98399	1	$R.L$	-0.288955
1.73601	1	$R.R.L.L$	-0.064746
2.13111	2	$R.L.R.L.L$	-0.069526
2.66193	2	$R.L.R.R.L.L$	-0.032848
2.89815	2	$R.L.L.R.R.L.L$	-0.024028
3.15482	2	$R.L.R.L.R.L.L$	-0.017289
3.54271	1	$R.L.R.R.L.R.L.L$	-0.0053429
3.62732	2	$R.L.R.L.R.R.L.L$	-0.0096416
3.80470	2	$R.L.R.R.L.R.R.L.L$	-0.0077879

Continued

$\ell_\gamma \approx$	$s(\gamma)$	γ	$A(\gamma) \approx$
3.93595	2	$R.L.R.L.L.R.R.L.L$	-0.0066608
4.15197	2	$R.L.R.L.R.L.R.L.L$	-0.0051635
4.20181	1	$R.L.L.R.R.L.R.R.L.L$	-0.0024355
4.39146	2	$R.L.R.R.L.L.R.R.L.L$	-0.0039068
4.48926	2	$R.L.R.L.R.R.L.R.L.L$	-0.0034894
4.60473	2	$R.L.R.L.R.L.R.R.L.L$	-0.0030555
4.65401	2	$R.L.L.R.R.L.L.R.R.L.L$	-0.0028877
4.76043	2	$R.L.R.L.R.R.L.R.R.L.L$	-0.0025571
4.84180	4	$R.L.R.L.L.R.L.R.R.L.L$	-0.0046617
4.93876	2	$R.L.R.L.R.L.L.R.R.L.L$	-0.0020879
5.01322	2	$R.L.R.L.L.R.L.L.R.R.L.L$	-0.0019192
5.14068	2	$R.L.R.L.R.L.R.L.R.L.L$	-0.0016622

Continued

$\ell_\gamma \approx$	$s(\gamma)$	γ	$A(\gamma) \approx$
5.20802	2	$R.L.R.L.L.R.R.L.R.R.L.L$	-0.0015409
5.28890	2	$R.L.R.L.R.L.L.R.L.R.L.L$	-0.0014072
5.28890	2	$R.L.R.R.L.R.L.L.R.R.L.L$	-0.0014072
5.35146	2	$R.L.R.L.R.R.L.L.R.R.L.L$	-0.0013120
5.42680	1	$R.L.R.L.R.R.L.R.L.R.L.L$	-0.00060298
5.45943	2	$R.L.R.L.R.L.R.R.L.R.L.L$	-0.0011628

Table: The first several lengths of primitive hyperbolic closed geodesics of the $(2, 3, 7)$-triangle group orbifold together with their representations and their contribution to the Casimir energy.

Hyperbolic contribution from the first 50

Lemma

The contribution to the Casimir energy from the first 50 primitive geodesics rounded to six decimal places is equal to -0.5680851.

What about the rest?

- $\mathcal{N}_L = \#\{\ell_\gamma \leq L\}$ the number of primitive hyperbolic geodesics γ (counted with multiplicity) of length ℓ_γ less or equal than L.
- Prime geodesic theorem says $\lim_{L\to\infty} \frac{\mathcal{N}_L(L)}{e^L} = 1$.
- Enumerate lengths (counting multiplicity) $\ell_j \implies$

$$\lim_{j\to\infty} \frac{\ell_j}{\log(j) + \log\log(j)} = 1.$$

- For all $j < 51$ the inequality holds: $\ell_j \geq \log(j) + \log\log(j)$.

Good things come in 3s

$$B_1 = \sum_{j=51}^{10^7} \frac{1}{4\pi} \operatorname{csch}\left(\frac{\ell_j}{2}\right) K_1\left(\frac{\ell_j}{2}\right),$$

$$B_2 = \sum_{j=10^7+1}^{\infty} \frac{1}{4\pi} \operatorname{csch}\left(\frac{\ell_j}{2}\right) K_1\left(\frac{\ell_j}{2}\right),$$

$$B_3 = \sum_{n=2}^{\infty} \sum_{j=51}^{\infty} \frac{1}{4\pi n} \operatorname{csch}\left(\frac{n\ell_j}{2}\right) K_1\left(\frac{n\ell_j}{2}\right).$$

The estimate of B_1

- csch and K_1 are decreasing on $(0, \infty)$.
- Thus $\operatorname{csch}\left(\frac{n\ell_j}{2}\right) K_1\left(\frac{n\ell_j}{2}\right) \leq$
  $\operatorname{csch}\left(\frac{n(\log j + \log\log j)}{2}\right) K_1\left(\frac{n(\log j + \log\log j)}{2}\right)$.
- We use this to obtain the explicit estimate $B_1 \leq 0.138415$.

Technical lemma and some facts

- We prove that for all $j \geq 16$ and $n \in \mathbb{N}$

$$j^n \log^{n+1/2} j \leq (j^n \log^n j - 1)(\log j + \log\log j)^{1/2}.$$

- By G&R for any $j \geq 10^7$ we may replace K_1 by $K_{\frac{3}{2}}$.
- Observe:

$$\operatorname{csch}(z) K_{3/2}(z) = \sqrt{\frac{2\pi}{z}} \frac{1+z}{ze^{2z} - z}.$$

The estimates of B_2 and B_3

- Setting $z = n(\log j + \log\log j)/2$ and dividing by $4\pi n$

$$\frac{\operatorname{csch}\left[\frac{\log(j)+\log\log(j)}{2}n\right] K_{\frac{3}{2}}\left[\frac{\log(j)+\log\log(j)}{2}n\right]}{4\pi n} = \frac{n(\log(j)+\log\log(j))+2}{2\sqrt{\pi}n\,(j^n\log^n(j)-1)\,n^{3/2}(\log(j)+\log\log(j))^{3/2}}. \qquad (2)$$

- With a bit more calculations (including the good old integral-sum comparison from calculus!) we obtain $B_2 \leq 0.155402$ and $B_3 \leq 0.000224$.

Hyperbolic tail estimate

We therefore obtain the tail estimate for all but the first 50 hyperbolic elements.

Lemma

Under the assumption that $\ell_j \geq \log(j) + \log\log(j)$ *holds also for* $j \geq 51$*, the contribution from all but the first 50 hyperbolic elements is greater than or equal to -0.293867.*

Summing the terms

- The contribution of the elliptic elements is, up to six decimal places, 0.875676.
- The identity contribution to the Casimir energy is at least -0.0022.
- The contribution from the first 50 primitive hyperbolic geodesics is, up to six decimal places, -0.5680851.
- The contribution from all but the first 50 hyperbolic elements is at least -0.293867.
- Can we engage (warp travel) or not? Please go to menti.com 5404 0478.

Merci!